International Journal on Teaching and Learning Mathematics
2021, Vol. 1, No. 1, pp. 36-44
P-ISSN: 2621-2188, E-ISSN: 2621-2196

Senior high school students' argumentation in proving mathematical induction based on mathematical abilities

Natasya Ziana Walidah[1], Elly Susanti[1]

[1]Universitas Islam Negeri Maulana Malik Ibrahim, Jalan Gajayana 50 Malang, Indonesia

ARTICLE INFO
Original Article
doi: 10.18860/ijtlm.v4i1.14315

Keywords:
Argumentation, Proving Mathematical Induction, Mathematical Abilities

ABSTRACT
Argumentation and proof are two interrelated elements in mathematics, which is one of the important goals in mathematics education. Furthermore, these also need to be supported by students' mathematical abilities, so that it has implications for proving they did. This qualitative research will be explained descriptively which aims to find out senior high school students' argumentation in proving mathematical induction. This research subjects consisted of six senior high school students in Surabaya who had low, medium, and high mathematical abilities. Research data were collected through a written test about proving mathematical induction. Then, the data analysis will be carried out, including: sorting the data, presenting the data, and making conclusions. The results shows that senior high school students who have low mathematical abilities can proving mathematical induction which only bring up *claim* and *evidence* in their argumentation. Meanwhile, senior high school students who have medium and high mathematical abilities can proving mathematical induction which bring up *claim*, *evidence*, and *reasoning* in their argumentation.

*Corresponding author.
E-mail: ellysusanti@mat.uin-malang.ac.id

How to cite: Walidah, N. Z., & Susanti, E. (2021). Senior high school students' argumentation in proving mathematical induction based on mathematical abilities. *International Journal on Teaching and Learning Mathematics*, *4*(1), 36-44.

1. INTRODUCTION

Argumentation have been described by various studies (Brown, 2017; Cervantes-Barraza et al., 2020; Suhendra, 2015; Walidah et al., 2019; Zambak & Magiera, 2020) and become one of the important goals that will or want to be achieved in mathematics education. Argumentation is defined as a statement constructed by students individually with accompanied by logical and relevant *evidence* (Ruggiero, 2014). In other words, when students are giving their argumentation, these an argument or idea will be generated to convince others regarding the truth of their statement or conclusion that they have been made (Aberdein, 2009).

Argumentation which built by students are also related to the components that make it up. This study uses McNeill & Krajcik's argumentation components which have been used in several studies (González-Howard et al., 2017; González-Howard & McNeill, 2019; McNeill et al., 2017; McNeill & Krajcik, 2012; Mikeska & Howell, 2020). These results were effective to highlight students' argumentation as a form of knowledge production to make conclusion. These components, include: 1) *claim*, is a statement that functions as an answer to a question, problem, or phenomenon; 2) *evidence*, is a statement that used to answer questions, solve problems, or make decisions, so that it can support the existence of a *claim*; 3) *reasoning*, is a justification that gives strength to the *evidence* in supporting a *claim*; and 4) *rebuttal* is a statement that used to explain other alternatives or the suitability of a *claim* (McNeill, 2011; Mcneill & Krajcik, 2008; McNeill & Martin, 2011).

Meanwhile, when students are giving their argumentation, they also do mathematical activities, such as: make conjectures, provide justification, and make conclusions (Knudsen et al., 2014). For example, when students make conjectures, they must provide justification using relevant and logical *evidence* against their conjectures, so that the conclusions can be made. In line with this, according to Toulmin (2003), that: 1) argumentation and proof are considered as rational justifications; 2) argumentation and proof are served to convince; 3) argumentation and proof are presented to the general public; and 4) argumentation and proof are became a “field”. It means, argumentation and proof can be said to be an activity have an important role in proving mathematical statement and is a part of mathematical proof.

The relationship between argumentation and proof were found in mathematical proof (Hapipi et al., 2019; Laamena, 2017). Mathematical proof is a fundamental part in mathematics (CadwalladerOlsker, 2011), it also the process of building a set of arguments correctly and logically so that related according to the rules of inference which aims to validate the truth of a mathematical statement (Firmasari & Sulaiman, 2019; Kartini & Suanto, 2015). Various methods of mathematical proof have been introduced in secondary education and will be discussed more in higher education (Firmasari & Sulaiman, 2019), which one in proving mathematical induction.

Mathematical induction is a method of proving mathematical induction that can be used to prove the truth of a statement that applies to all natural numbers and positive integers from several variables (Firmasari & Sulaiman, 2019; Hine, 2017). The steps in proving mathematical induction (Michaelson, 2008), include: 1) *the initial step*, such as: "Show that $P(n)$ is true where P is the identity to be proven and n is the first natural number whose identity is true" ; and 2) *the inductive step*, such as: "If $P(k)$ is true for every $k \geq n$, then show $P(k+1)$ is true".

In addition, not only argumentation in proving mathematical induction, but students' mathematical abilities are also needed which may have implications for both of them. Mathematical abilities are often referred to the ability when student uses to solve mathematical problems, as well as the ability to obtain, process, and save the mathematical concept (Karsenty, 2014) that give new mathematical understandings and skills for them. Nizoloman (2013) states that mathematical abilities are related to the process of using or manipulating numbers effectively in solving mathematical problems, which can help students to find various alternative solutions. This is closely related to the diversity of arguments that students produce when they do argumentation, then they used in mathematical proof. The mathematical abilities in this study are the results of test that consist of low, medium, and high mathematical abilities (Pungkasari et al., 2020).

The descriptions show that mathematical abilities can underlie the existence of students' argumentation in mathematical proof. This means that through the existence of different levels of students' mathematical abilities, it can be seen how argumentation generated by students are used as part of mathematical proofs, in this study is proving mathematical induction. Therefore, this article to examine and explain descriptively about students' argumentation who have low, medium, and high mathematical abilities in proving mathematical induction.

International Journal on Teaching and Learning Mathematics
2021, Vol. 1, No. 1, pp. 36-44
P-ISSN: 2621-2188, E-ISSN: 2621-2196

2. METHOD

This research is a qualitative research with descriptive explanation (Miles et al., 2014) which aims to describe senior high school students' argumentation in proving mathematical induction based on mathematical abilities. The subjects were six senior high school students in Surabaya who had been given the test of mathematical abilities on compulsory mathematics material for tenth grade in odd semester. The subjects were also grouped into three levels of mathematical abilities using the fixed comparison method to triangulate the research data sources, namely two students have low mathematical abilities with their score less than 70, two students have medium mathematical abilities with their score equal to 70 up to less than 85, and two students have high mathematical abilities with their score more than or equal to 85. The subjects that have been categorized were mention such as in the following table below.

Table 1. The subjects of this research based on students' mathematical abilities

Initial	Score	The Category of Mathematical Abilities	Code
AW	57	Low	L1
US	61	Low	L2
AR	78	Medium	M1
DW	83	Medium	M2
PA	88	High	H1
MV	95	High	H2

Meanwhile, the research data was collected through a written test which contains the problems in proving mathematical induction about sequence and series at the third level. Then, the data will be analyzed according to Miles et al. (2014), including: 1) Condensation of data by sorting data from the results of the test of proving mathematical induction that according to the focus of research and the needs of the researcher; 2) Presentation of data to understand and analyze data in depth related to senior high school students' argumentation in proving mathematical induction using data that has gone through the data condensation stage and refers to McNeill & Krajcik's (2012) argumentation components; and 3) Make conclusions in accordance with the theory used in this study.

3. RESULTS AND DISCUSSION

Based on the research that has been done, it shows that six subjects who are divided into three categories of mathematical abilities have differences way in generating their argumentation in proving mathematical induction. The difference can be seen in the completeness of the initial step and the inductive step which they made.

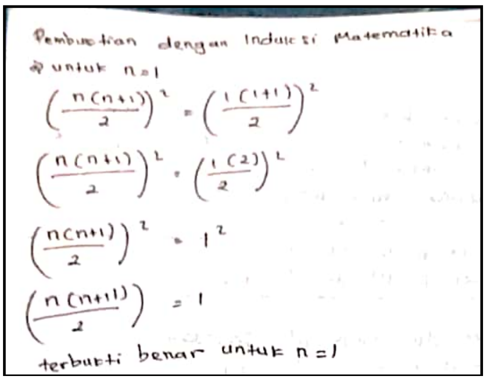


Figure 1. The initial step in proving mathematical induction of L1

International Journal on Teaching and Learning Mathematics
2021, Vol. 1, No. 1, pp. 36-44
P-ISSN: 2621-2188, E-ISSN: 2621-2196

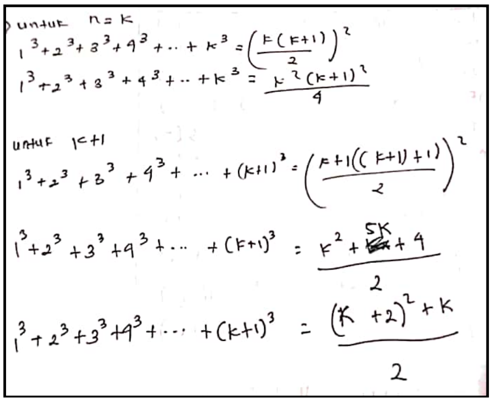


Figure 2. The inductive step in proving mathematical induction of L1

Based on Figure 1 and Figure 2, it can be seen that L1 can write down the initial step and the inductive step in proving mathematical induction from the given problem. Figure 1 shows that L1 can provide *evidence* in the form of a statement on the *claim* is true for n = 1. However, L1 did not provide *reasoning* in the inductive step which caused the solution were produced did not complete, as in Figure 2. In addition, L1 also did not provide another alternative as a form of *rebuttal* to complete the argumentation regarding a *claim* which is mentioned in the given problem. This causes L1's argumentation to be fulfilled only *claim* and *evidence*.

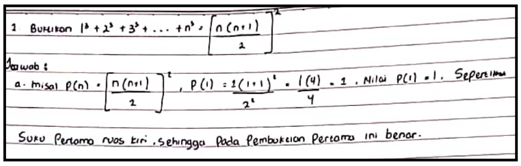


Figure 3. The initial step in proving mathematical induction of L2

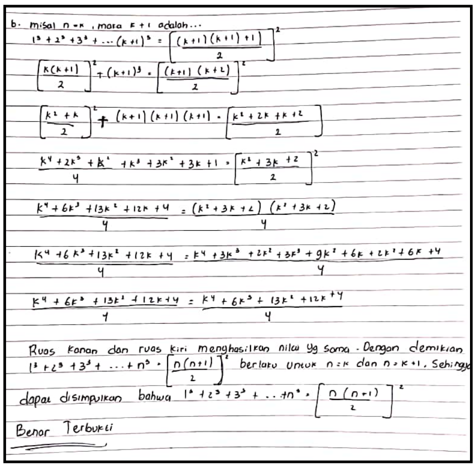
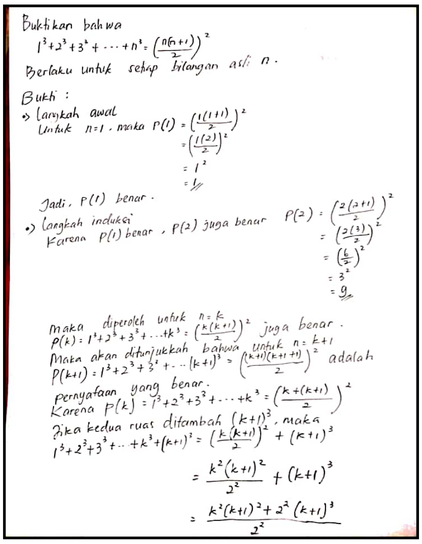


Figure 4. The inductive step in proving mathematical induction of L2

Figure 3 and Figure 4 show that L2 can solve the given problem by using the steps of proving mathematical induction, which can be seen from the initial step and the inductive step it produces. However, unlike L1, L2 can provide *evidence* and *reasoning* at the initial step (see Figure 3) and the inductive step (see Figure 4). Meanwhile, *rebuttal* was not given by L2 because the results of the given proof were in accordance with *claim* on the given problem. This causes argumentation which generated by L2 to be fulfilled in a *claim* on the given problem, *evidence*, and *reasoning*.

In the description above, both L1 and L2 who have low mathematical abilities in proving mathematical induction do the various solutions. Both of them can provide *evidence*, but *reasoning* is only given by L2.

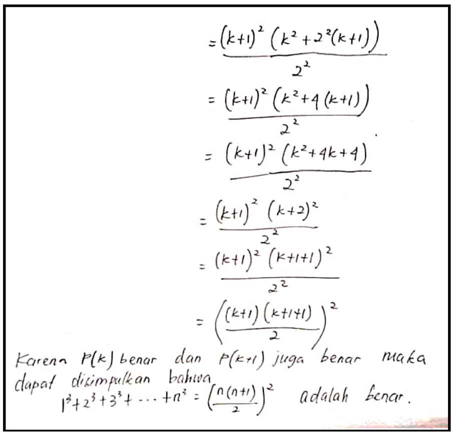


Buktikan bahwa
$1^3+2^3+3^3+\dots+n^3 = \left(\frac{n(n+1)}{2}\right)^2$
Berlaku untuk setiap bilangan asli n.

Bukti :
• Langkah awal
Untuk $n=1$, maka $P(1) = \left(\frac{1(1+1)}{2}\right)^2 = \left(\frac{1(2)}{2}\right)^2 = 1^2 = 1$
Jadi, $P(1)$ benar.
• Langkah induksi
Karena $P(1)$ benar, $P(2)$ juga benar $P(2) = \left(\frac{2(2+1)}{2}\right)^2 = \left(\frac{2(3)}{2}\right)^2 = \left(\frac{6}{2}\right)^2 = 3^2 = 9$

Maka diperoleh untuk $n=k$
$P(k) = 1^3+2^3+3^3+\dots+k^3 = \left(\frac{k(k+1)}{2}\right)^2$ juga benar.
Maka akan ditunjukkah bahwa untuk $n=k+1$
$P(k+1) = 1^3+2^3+3^3+\dots+(k+1)^3 = \left(\frac{(k+1)(k+1+1)}{2}\right)^2$ adalah pernyataan yang benar.
Karena $P(k) = 1^3+2^3+3^3+\dots+k^3 = \left(\frac{k+(k+1)}{2}\right)^2$
Jika kedua ruas ditambah $(k+1)^3$, maka
$1^3+2^3+3^3+\dots+k^3+(k+1)^3 = \left(\frac{k(k+1)}{2}\right)^2 + (k+1)^3$
$= \frac{k^2(k+1)^2}{2^2} + (k+1)^3$
$= \frac{k^2(k+1)^2+2^2(k+1)^3}{2^2}$

Figure 5. The initial step in proving mathematical induction of M1

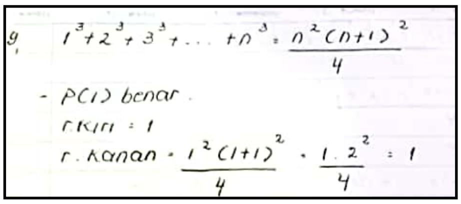


$= \frac{(k+1)^2\left(k^2+2^2(k+1)\right)}{2^2}$
$= \frac{(k+1)^2\left(k^2+4(k+1)\right)}{2^2}$
$= \frac{(k+1)^2\left(k^2+4k+4\right)}{2^2}$
$= \frac{(k+1)^2(k+2)^2}{2^2}$
$= \frac{(k+1)^2(k+1+1)^2}{2^2}$
$= \left(\frac{(k+1)(k+1+1)}{2}\right)^2$

Karena $P(k)$ benar dan $P(k+1)$ juga benar maka dapat disimpulkan bahwa
$1^3+2^3+3^3+\dots+n^3 = \left(\frac{n(n+1)}{2}\right)^2$ adalah benar.

Figure 6. The inductive step in proving mathematical induction of M1

In the Figure 5 and Figure 6 show the M1's argumentation when proving mathematical induction with the initial step and the inductive step, it causes all three components of the argumentation to be fulfilled. Both of them are *claim*, *evidence*, and *reasoning*. One component that does not appear is *rebuttal* because the *evidence* produced by M1 is in accordance with *claim* on the given problem.



9, $1^3+2^3+3^3+\dots+n^3 = \frac{n^2(n+1)^2}{4}$
- $P(1)$ benar.
r. kiri $= 1$
r. kanan $= \frac{1^2(1+1)^2}{4} = \frac{1.2^2}{4} = 1$

Figure 7. The initial step in proving mathematical induction of M2

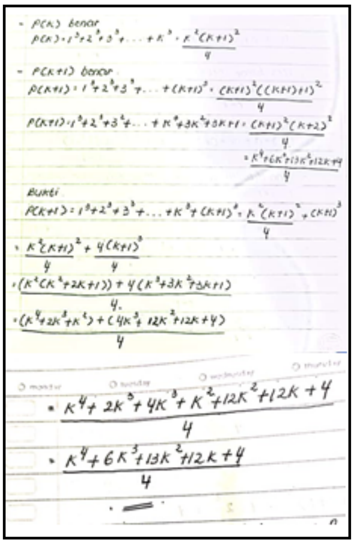


Figure 8. The inductive step in proving mathematical induction of M2

In Figure 7 and Figure 8 above, it can be seen that M2 can solve the given problem through the steps in proving mathematical induction. Figure 7 shows that M2 is able to solve the problem at the initial step by bringing up *evidence*. Then, Figure 8 shows that M2 also provides *evidence* and *reasoning* in the inductive step of mathematical induction. This means that the components of the argumentation produced by M2, namely *evidence* and *reasoning*, can support and prove the truth of a *claim* in the given problem.

The description above shows that both students who fall into the category of medium mathematical abilities, namely M1 and M2, do the initial step and the inductive step correctly. This is because both can build *evidence* and *reasoning* that supports the existence of a *claim* on the given problem.

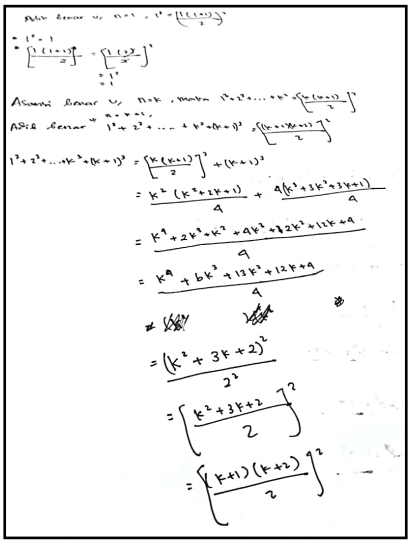


Figure 9. The initial and the inductive step in proving mathematical induction of H1

In Figure 9, it shows that H1 uses the initial step and the inductive step in proving mathematical induction. This causes H1 to be able to bring up the argumentation's components, namely *evidence* and *reasoning*, which support the truth of a *claim* in the given problem.

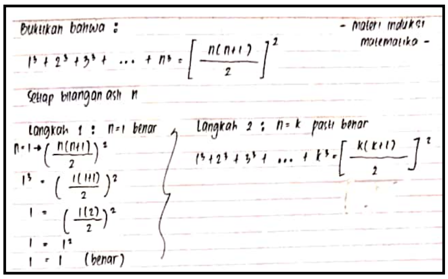


Figure 10. The initial step in proving mathematical induction of H2

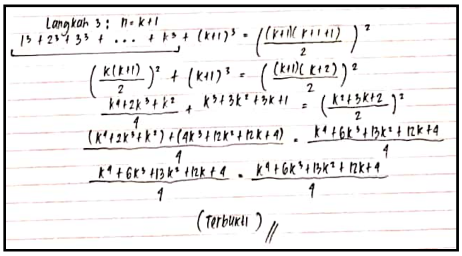


Figure 11. The inductive step in proving mathematical induction of H2

Based on the completion of H2 in proving mathematical induction as shown in Figure 10 and Figure 11 above, it shows that H2 can perform the initial step (see Figure 10) and the inductive step correctly (see Figure 11). In addition, H2 can also produce *evidence* and *reasoning* that supports a *claim* on the given problem, as shown in both figure. In other words, the argumentation which generated by H2 are based on the components of *evidence* and *reasoning*.

From both senior high school students who fall into the category of high mathematical abilities, namely H1 and H2, it shows that they're proving mathematical induction with the complete and correct ways. This is because both of them can build *evidence* and *reasoning* that supports the existence of a *claim* on the given problem.

Thus, it is seen that argumentation plays a role in producing appropriate *evidence* in proof (Hapipi et al., 2019; Laamena, 2017), especially in proving mathematical induction. This can be seen from the initial step and the inductive step which is produced by all subjects (Michaelson, 2008) which are supported by McNeill & Krajcik's (2012) argumentation components, namely *claim*, *evidence*, and *reasoning*. In addition, it is also seen that the mathematical abilities of all subjects have an effect on bringing up appropriate solutions (Nizoloman, 2013) with the steps in proving mathematical induction.

Furthermore, it is also seen that senior high school students with medium and high mathematical abilities can generate *evidence* and *reasoning* in proving mathematical induction. However, senior high school students with low mathematical abilities can only produce *evidence* in proving mathematical induction. Meanwhile, *rebuttal* was not generated because the results obtained by all subjects did not contradiction with a *claim* which given to the problem.

4. CONCLUSION

We can conclude that senior high school students who have low mathematical ability can prove mathematical induction. Both of them were bringing up *claim* and *evidence* for the initial step, but one was bringing up *reasoning* for the inductive step. It can be said that they propensity both *claim* and *evidence* of the argumentation components which were representing their argumentation. Meanwhile, senior high school students who have medium and high mathematical abilities can also prove mathematical induction with their argumentation. Both on the initial and inductive step, they were bringing up *claim*, *evidence*, and *reasoning*. So that, it can be said that they propensity in all argumentation components which were representing their argumentation.

ACKNOWLEDGMENTS

We appropriate thank to senior high school students in Surabaya who have been willing to our research subjects. Because of their contributions was successfully support the completion of this research.

REFERENCES

Aberdein, A. (2009). Mathematics and argumentation. *Foundations of Science*, *14*(1–2), 1. https://doi.org/https://doi.org/10.1007/s10699-008-9158-3

Brown, R. (2017). Using collective argumentation to engage students in a primary mathematics classroom. *Mathematics Education Research Journal*, *29*(2), 183–199. https://doi.org/10.1007/s13394-017-0198-2

CadwalladerOlsker, T. (2011). What do we mean by mathematical proof? *Journal of Humanistic Mathematics*, *1*(1), 33–60. https://doi.org/10.5642/jhummath.201101.04

Cervantes-Barraza, J. A., Hernandez Moreno, A., & Rumsey, C. (2020). Promoting mathematical proof from collective argumentation in primary school. *School Science and Mathematics*, *120*(1), 4–14. https://doi.org/10.1111/ssm.12379

Firmasari, S., & Sulaiman, H. (2019). Kemampuan pembuktian matematis mahasiswa menggunakan induksi matematika. *Journal Of Medives: Journal Of Mathematics Education IKIP Veteran Semarang*, *3*(1), 1–9. https://doi.org/https://doi.org/10.31331/medivesveteran.v3i1.642

González-Howard, M., & McNeill, K. L. (2019). Teachers' framing of argumentation goals: Working together to develop individual versus communal understanding. *Journal of Research in Science Teaching*, *56*(6), 821–844. https://doi.org/10.1002/tea.21530

González-Howard, M., McNeill, K. L., Marco-Bujosa, L. M., & Proctor, C. P. (2017). 'Does it answer the question or is it french fries?': An exploration of language supports for scientific argumentation. *International Journal of Science Education*, *39*(5), 528–547. https://doi.org/10.1080/09500693.2017.1294785

Hapipi, H., Azmi, S., & Turmuzi, M. (2019). Profil argumentasi mahasiswa dalam melakukan pembuktian pada topik analisis kompleks. *Mandalika Mathematics and Educations Journal*, *1*(1), 41–52. https://doi.org/10.29303/jm.v1i1.1243

Hine, G. (2017). Proof by mathematical induction: Professional practice for secondary teachers. *26th Biennial Conference of the Australian Association of Mathematics Teachers*, 1–8.

Karsenty, R. (2014). Mathematical Ability. In *Encyclopedia of Mathematics Education*. https://doi.org/10.1007/978-94-007-4978-8_94

Kartini, K., & Suanto, E. (2015). Analisa kesulitan pembuktian matematis mahasiswa pada mata kuliah analisis real. *SEMIRATA 2015 Bidang MIPA BKS-PTN Barat*, *1*(1), 189–199.

Knudsen, J., Lara-meloy, T., Stevens, H. S., & Rutstein, D. W. (2014). Advice for mathematical argumentation. *Mathematics Teaching in the Middle School*, *19*(8), 494–500. https://doi.org/https://doi.org/10.5951/mathteacmiddscho.19.8.0494

Laamena, C. M. (2017). Karakteristik warrant dalam argumentasi dan pembuktian matematis. *Seminar Nasional Hasil Penelitian Universitas Kanjuruhan Malang*, 296–302.

McNeill, K. L. (2011). Elementary students' views of explanation, argumentation, and evidence, and their abilities to construct arguments over the school year. *Journal of Research in Science Teaching*. https://doi.org/10.1002/tea.20430

McNeill, K. L., GonzÁlez-Howard, M., Katsh-Singer, R., & Loper, S. (2017). Moving beyond pseudoargumentation: Teachers' enactments of an educative science curriculum focused on argumentation. *Science Education*, *101*(3), 426–457. https://doi.org/10.1002/sce.21274

Mcneill, K. L., & Krajcik, J. (2008). Inquiry and scientific explanations: Helping students use evidence and reasoning. In *J. Luft, R. Bell, & J. Gess-Newsome (Eds.), Science as inquiry in the secondary setting* (pp. 121–134). Arlington, VA: National Science Teachers Association Press.

McNeill, K. L., & Krajcik, J. S. (2012). *Supporting grade 5–8 students in constructing explanations in science: The claim, evidence and reasoning framework for talk and writing*. New York, NY: Pearson Allyn & Bacon.

McNeill, K. L., & Martin, D. M. (2011). Claims, evidence, and reasoning: Demystifying data during a unit on simple machines. *Science and Children*, *48*(8), 52–56. http://searkscience.pbworks.com/w/file/fetch/70117336/2-Claimsevidence.pdf

Michaelson, M. T. (2008). A literature review of pedagogical research on mathematical induction. *Australian Senior Mathematics Journal*, *22*(2), 57–62. https://doi.org/https://search.informit.org/doi/10.3316/informit.410329014071111

Mikeska, J. N., & Howell, H. (2020). Simulations as practice-based spaces to support elementary teachers in learning how to facilitate argumentation-focused science discussions. *Journal of Research in Science Teaching*, *57*(9), 1356–1399. https://doi.org/10.1002/tea.21659

Miles, M. B., Huberman, A. M., & Saldana, J. (2014). Q*ualitative data analysis: A methods sourcebook*. Sage Publications.

Nizoloman, O. N. (2013). Relationship between mathematical ability and achievement in mathematics among female secondary school students in Bayelsa State Nigeria. *Procedia - Social and Behavioral Sciences*, *106*. https://doi.org/10.1016/j.sbspro.2013.12.254

Pungkasari, A. M., Purwosetiyono, F. X. D., & Pramasdyahsari, A. S. (2020). Kemampuan spasial perception dalam menyelesaikan masalah geometri berdasarkan Teori Van Hiele ditinjau dari kemampuan matematika. *Prismatika: Jurnal Pendidikan Dan Riset Matematika*, *3*(1), 75–86. https://doi.org/https://doi.org/10.33503/prismatika.v3i1.1123

Ruggiero, V. R. (2014). *Becoming a critical thinker*. USA: Cengage Learning.

Suhendra, M. (2015). Argumentasi matematik sebagai sebuah kompetensi matematik. *Jurnal Pengajaran Matematika dan Ilmu Pengetahuan Alam*, *15*(1). https://doi.org/10.18269/jpmipa.v15i1.284

Toulmin, S. E. (2003). The uses of argument: Updated edition. In *The Uses of Argument: Updated Edition*. https://doi.org/10.1017/CBO9780511840005

Walidah, N. Z., Kusaeri, K., & Yudi, U. (2019). Mengoptimalkan dukungan guru dalam melibatkan siswa menggunakan argumentasi kolektif pada pembelajaran matematika. *MaPan*, *7*(2), 261–280. https://doi.org/10.24252/mapan.2019v7n2a7

Zambak, V. S., & Magiera, M. T. (2020). Supporting grades 1–8 pre-service teachers' argumentation skills: constructing mathematical arguments in situations that facilitate analyzing cases. *International Journal of Mathematical Education in Science and Technology*, *51*(8). https://doi.org/10.1080/0020739X.2020.1762938