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Describing the phenomena of students' representation in solving ill-posed and well-posed problems

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A B S T R A C T
Mathematical representation is an essential aspect of mathematical problem-solving. But students' ability of an accurate representation in ill-posed problem-solving is still very minimal compared to that in well-posed problem-solving. However, ill-posed problem supported mathematical abstraction used in mathematical concept understanding. This study described the representations used by mathematics education students in solving ill-posed and well-posed problems. Thirty Indonesian matematics education students have solved ill-well posed problems by using think-aloud. Researchers also collected data using a video recorder and a field note. Data were analyzed by a constant comparative method so that it was obtained the different characteristics of representations between solving ill-posed and well-posed problems. The finding of the study showed that verbal and symbolic representations were used by subjects to compute, detect, and correct error. They also justified their answers in ill-posed problem-solving. However, the visual representation was only used by first subject to identify and correct error. The subjects lacked to expose necessary information to solve the ill-posed problem compared to the well-posed problem.

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1. INTRODUCTION

Representation is the disclosure of mathematical ideas by using various means such as spoken language, written language, symbols, pictures, diagrams, models, charts, or using physical objects (Caglayan & Olive, 2010; Goldin-Meadow & Beilock, 2010; Hwang et al., 2009). The use of a mathematical representation is an main modality in expressing mathematical thinking (NCTM, 2000). Representation is widely used to learn and reinforce students' understanding of mathematical concepts and mathematical problems (Bal, 2014; De Bock et al., 2015; Merritt et al., 2017; Rau, 2017). Mathematical representation is not only process-oriented but also provides a precious product. So in the mathematics learning process, mathematical representation is one aspect that must be prioritized. By students' mathematical representation accurately, they can expand their capabilities in solving mathematical problems.

Mathematical representation becomes an interesting topic to be studied by several researchers of mathematics education. Roche and Clarke (2013) investigated primary teachers' representations in the division problems. David et al. (2014) identified visual representations in the algebra classrooms. Stylianou and Silver (2004) examined visual representations in advanced mathematical problem-solving. Adu-Gyamfi et al. (2012) inquired translation errors associated with mathematical representations. Xun and Land (2004) made a framework for scaffolding ill-Structured problem-solving processes using question prompts and peer interactions. Those studies do not explore the phenomena of students' representation in the ill-posed and well-posed problems.

Mathematical representation is a crucial concept of the cognitive process. Mathematical representation supplements student learning because it provides concrete ways for students to approach mathematics concepts and make connections between their experiences and symbolic mathematics. Researches related to the importance of representation in mathematics were performed by some scholars (Bal, 2014; Birgin, 2012; Caglayan & Olive, 2010; Villegas et al., 2009). Those studies found that the ability of student representation is the key to success in understanding mathematical concepts and problem-solving. The representation of a problem consists principally of the solver's interpretation of the problem, which finds how easily a problem can be solved. The solver extracts information and attempts to understand the problem or connect it to existing knowledge to form an integrated representation. If schema can be activated during problem representation, the solution process is schema driven, with a little search for solution procedures. If appropriate schema cannot be activated, the problem solver goes back to an earlier stage and redefines the problem or uses another strategy to solve the problem.

Based on the importance of mathematical representation in problem-solving, one of the problems that support multiple representations is ill-structure problems. Through solving the ill-structured problem, students can improve their mathematical abstraction capability. The Ill-structured problems are the kinds of problem that are conflicting goals, multiple solution methods, unanticipated problems, distributed knowledge, collaborative activity systems, and multiple forms of problem representation (Jonassen, 1997, 2011). Thus, using the ill-structured problem will support students' mathematical representation ability. This is related to Jonassen (2011) that defined ill-structured problems, contents authenticity, complexity, and openness. The complexities made multiple problem representations.

The ill-structured problem also includes a real word that expects students to define the problems as well as determine the information and skills needed to solve them. Ill-structured problem is like that someone encounters in everyday life, in which one or several characteristics of the situation is not well specified, the goals are unclear, and there is insufficient information to solve the problem (Chi & Glaser, 1985; Reed, 2016; Sinnott, 1989; Voss & Post, 1988). The Ill-structured problems should contain the situation of daily life and be relevant enough to deduce an integral part of the actual condition. The complexity of the ill-structured problem can pose difficulties for students to organize the problem and monitor the solution process (Feltovich et al., 1996; Ge & Land, 2003). Students often execute misconceptions or superficial conceptions, which can impede their success in solving problems (Feltovich et al., 1996).

The aspects of conflicting goals and multiple solution methods in ill-structured problems were seen on the ill-posed problem. Hadamard (1923) introduced Ill-posed and well-posed problems. He defined ill-posed problem as a problem that either has no solutions in the desired class or has many solutions, or the solution procedure is unstable. The solution instability causes the most difficulties in solving ill-posed problems. Therefore, the term "ill-posed problems" is often used for unstable problems (Kabanikhin, 2008). The inverse of ill-posed problems is a well-posed problem. Well-posed problems are the problem that has a unique solution to the problem that continuously depends on its data (Kabanikhin, 2008). Well-posed problems can be called a closed-ended problem, well-defined problem, or well-structured problem, whereas ill-posed problem is also called an open-ended problem, ill-defined problem, or ill-structured problem.

The purpose of this study was to describe how students use mathematical representations in solving well-posed and ill-posed problems. Educators can use the result of the study as guiding in developing learning device or manipulative media to increase students' mathematical representation ability. We developed an indicator based on Villegas et al. (2009) framework for analyzing student's mathematical representation as shown in Table 1. To describe the mathematical representation, we took into account the following types of external representations: 1) verbal representation of the well-posed problems and ill-posed problems: consisting fundamentally of the well-posed problems and ill-posed problems as stated, whether in writing or spoken (Vb); 2) visual representation: consisting of drawings, diagrams or graphs, as well as any kind of related action (Vs); 3) symbolic representation, consisting of numbers, operation and relation signs; algebraic symbols, and any kind of action referring to these (Sb); and 4) their translation among multiple representation in solving ill-posed problem and well-posed problem (Vb-Vs, Vb-Sb, Vs-Sb).

Table 1. Framework for mathematical representation analysis

Part 1: Verbal representation of well-posed and ill-posed problems solving (Vb)
Description: student finds the problem. Indicators: 1) student reads the well-ill posed problem aloud or silently, and 2) student verbalizes the well-ill posed problem used usual style of talk.
Part2: Visual representation of well-posed and ill-posed problems solving (Vs)
Description: student uses or modifies visual representations. Indicators: 1) student draws problem with visual representation, or modifies such representations made earlier, 2) student operates formulas with visual representations, 3) student observes a visual representation, or verbalizes terms associated with visual representations, and 4) student uses the gesture to show visual representations.
Part3: Symbolic representation of well-posed and ill-posed problems solving (Sb)
Description: student produces, operates or modifies symbolic representations. Indicators: 1) student solves a symbolic expression, 2) student verbalizes how he/she can solve an equation, or checks how it was solved, and 3) student modifies or eliminates a symbolic expression.
Part 4: Translation between a verbal representation and visual representation of well-posed and ill-posed problems solving (Vb-Vs)
Description: student relates a visual representation to a verbal representation. Indicators: 1) student makes a visual representation directly from the well-posed and ill-posed problems, either without modifying it, 2) student transforms or modifies a visual representation according to a new interpretation of the well-posed and ill-posed problems, and 3) student establishes relationships between the well-posed and ill-posed problems and a visual representation, using verbalizations or gestures.
Part 5: Translation between a visual representation and symbolic representation of well-posed and ill-posed problems solving (Vs-Sb)
Description: student somehow relates a pictorial representation to a symbolic representation. Indicators: 1) student formulates on paper a symbolic expression or part of one based on a visual representation, or makes a visual representation based on symbolic expression, 2) student establishes relationships between a symbolic expression and a visual expression using verbalizations or gestures, 3) student makes changes or eliminates a visual representation made earlier, based on symbolic results obtained, and 4) student modifies or eliminates symbolic

representations due to the results obtained in visual representations or a new visual representation.

Part 6: Translation between symbolic representation and verbal representation of well-posed and ill-posed problems solving (Sb-Vb)

Description: student somehow relates a symbolic representation to a verbal representation.

Indicators: 1) student formulates a symbolic expression based on their interpretation, without modifying it, 2) student modifies a symbolic expression due to a new interpretation of the well-posed and ill-posed problems, and 3) student reformulates the well-posed and ill-posed problems in a new way due to some result obtained from a symbolic expression.

2. METHOD

This study used a qualitative approach. In qualitative research, the instruments are the researcher itself. Researcher takes part as a planner, a data collector, analyzer, and a person who conclude. In this case, researchers as instruments supported by other instruments among others is ill-well posed problems, think-aloud video recording, and field notes. The participants of the study were 30 students who took the teaching and learning courses in a private university, East Java, Indonesia.

The steps to obtain the data were: (1) the researchers gave ill-well posed problem to students. The problems in the form of a design problem for ill-posed problem and algebra problem for the well-posed problem as shown in Figure 1 and Figure 2, respectively, (2) subjects solve mathematical problems in 30 minutes to think-aloud. In the think-aloud, the student (participant) is asked to express aloud any words that her/his thinking at first receiving a problem to solve the problem. The researchers recorded the subject activity using a video recorder and recorded interesting things that happened on the field notes sheet, (3) the researchers reviewed the subject's answer sheet, video recordings, and field notes to know the activity of representation in ill-well posed problem-solving process, (4) researchers conducted transcription data after the whole data were obtained. The data transcribed were think-aloud recording data, (6) the researchers performed data reduction process by making abstraction in the form of a summary of core data, process, and statements that need to be maintained to be used as data, (7) researchers analyzed students' representation in ill-posed and well-posed problem-solving process using constant comparative method, (8) the researchers performed the triangulation process. This research used a technique triangulation process in the form of several uses of data collection techniques in the form of answer sheets of the subject in solving ill-well posed problems, think-aloud transcriptions, and field notes, (9) researchers drew conclusions through the obtained data.

Mr A will construct a new house. The design specifications of the home as follows. First, the house is a rectangular shape, 20m in width, and 10m in length, with a floor area of 200m^2. The house has five bedrooms, two bathrooms, a living room, a kitchen and dining area, a utility room and a balcony. The main bedroom has an area of 25m^2 with direct access to a bathroom. The other rooms have area sizes of 10-16 m^2. The living room, the kitchen and dining area have to be situated at the center. The living room must be 42m^2, and the kitchen and dining area is to be 20m^2. Try to develop the best design of this house.

Figure 1. Ill-posed problem

If $x \in R$ *and satisfying* $\frac{6}{1+\frac{4}{1+x}} = 2$. *Find the value of* x

Figure 2. Well-posed problem

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3. RESULTS AND DISCUSSION

In this article, we gave descriptions of S1 solutions. Figure 3 and Figure 5 demonstrate the differences of mathematical representation in ill-posed and well-posed problems of S1 solutions.

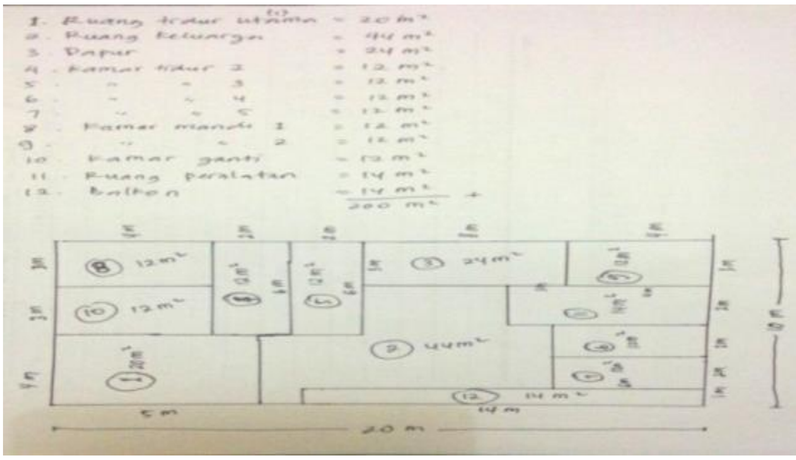

Figure 3. Subject S1's solution in ill-posed problem solving

Time	*Description*	*Type of Representations*
00:01	*Silent and see the problem.*	-
01:30	*The conditions are stated here ... the length is 20 meters and wide is 10 meters, and these are other specified sizes.*	*Vb-Sb, Sb-Vs*
01:42	*The first, I caught was that the door must be farthest from the main room. There are 5 rooms, one main bedroom ...emm each* $12\ m^2$ *wide.*	*Vb-Sb, Sb-Vs*
01:54	*Here on my first floor, there are 3 rooms. The first room is the main bedroom, the second room is* $6m \times 2m$*, the other one is* $2m \times 2m$*.*	*Vb-Sb, Sb-Vs*
02:03	*The other room is on the 2nd floor. The other room are* $4m \times 4m$*.*	*Vb-Sb, Sb-Vs*
02:33	*Earlier it was the bedroom, now the bathroom ... the bathroom here next to the main bedroom.*	*Vb*
02:41	*Whereas the living room must be in the middle, so I put it here.*	*Vb*
02:56	*Here is a dining room, next to the kitchen. The equipment room next to the kitchen, direct access.*	*Vb*
03:02	*Then I added more space to watching television and there were few other spaces.*	*Vb-Vs*
03:23	*And the balcony on the 2nd floor, I also put the garden near the balcony.*	*Vb-Vs*

Figure 4. S1's transcription data for ill-posed problem-solving

Figure 3 and Figure 4 indicate that S1 used verbal representation in arranging parts of rooms. S1 detected and corrected the error by visual representation with drawn the design. S1 used symbolic representation to verbalize area of each room. S1 made translation between a verbal representation to visual representation. S1 also translated visual representation to symbolic representation. S1 lacked to uncover necessary information about scale to compute and justified her answers.

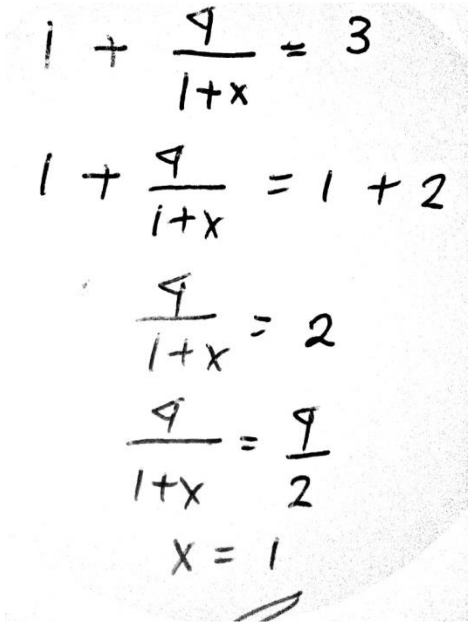

Figure 5. S1's solution in well-posed problem-solving

Time	*Description*	*Type of Representations*
00:01	*Silent and see the problem.*	-
00:35	*In the right side, I replace 3 by 1 plus 2, so* $\frac{4}{1+x}$ *equals to 2.*	*Vs-Sb*
01:06	*Then, in the right side, I replace again 2 with 4 divided by 2. So,* x *equals 1.*	*Vs-Sb*

Figure 6. S1's transcription data for well-posed problem-solving

Figure 5 and Figure 6 indicates that S1 used verbal representation to formulates a symbolic expression. S1 detected and corrected the error by symbolic representation with substitution $x = 1$ to the first equation. S1 transformed or modified a symbolic expression due to a new interpretation. S1 did translate between a visual representation to symbolic representation to compute and justified her answers.

Furthermore, we gave descriptions of S2 solutions. Figure 7 and Figure 9 show the differences of mathematical representation in ill-posed and well-posed problems of S2 solutions.

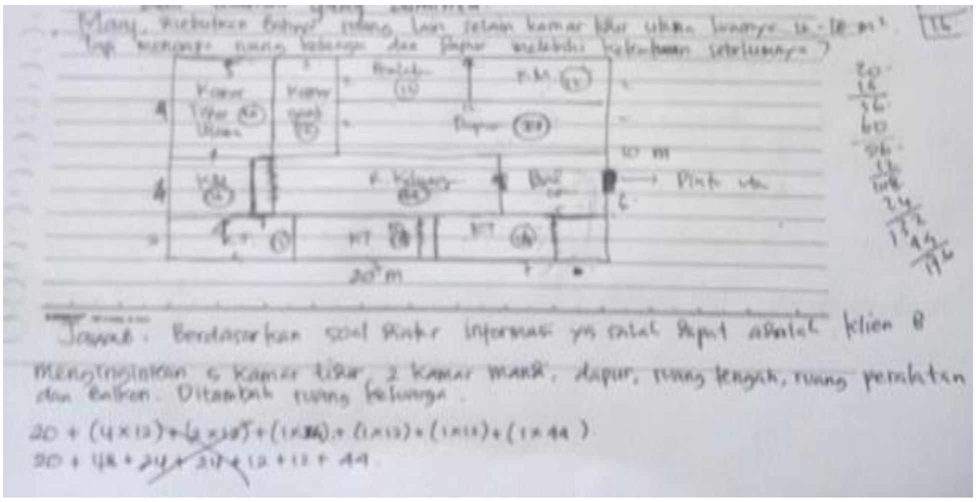

Figure 7.S2's solution in ill-posed problem-solving

Time	*Description*	*Type of Representations*
00:05	*After read the questions... the first, I listed are the spaces that must exist ...*	-
00:07	*So here (pointing at the picture of part 1) that is questioned there is stated that is the master bedroom of* $20\ m^2$*, then the main room of* $44\ m^2$*and the kitchen is* $24\ m^2$	*Vs-Sb, Sb-Vb*
00:12	*And then for another room, there are five bedrooms ... five of them I count, including the main bedroom.*	*Vb*
00:16	*Now in the matter, there is information ... for other spaces besides this rooms (pointing to part 1) it has a wide between* $12m^2$ *and* $16m^2$*. Well, here (pointing to the part 2) I specify the bedroom is the same for all the size is* $12\ m^2$*, bathroom* $12\ m^2$*, dressing room* $12m^2$*, equipment room* $14\ m^2$ *and balcony* $14\ m^2$*.*	*Vb-Sb, Sb-Vs, Vb-Vs*
00:52	*Then there is a position of bathrooms. I put the dressing room and bathroom here, near the main room. Well ... next., connected with the another room ... another room is* $12m^2$*. Now I put it here ...* *Ohh...Iso confused ma'am ... (He made the cross mark X on his work)*	*Vs-Vb, Sb-Vs, Sb*

Figure 8. S2's transcription data for ill-posed problem-solving

Figure 7 and Figure 8 show that S2 used visual representation to analyze the design of rooms. S2 used verbal representation to calculate the area of rooms. S2 used symbolic representation to verbalizes the area of each rooms. S2 translated a visual representation to verbal representation. S2 also made a translation from visual representation to symbolic representation. S2 lacked to uncover necessary information about scale to justified her answers.

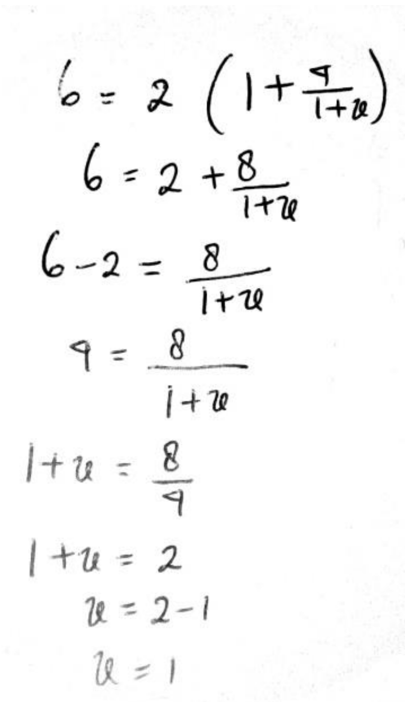

Figure 9. S2's solution in well-posed problem-solving

Time	*Description*	*Type of Representations*
00:27	*I multiplied both sides by* $(1+\frac{4}{1+x})$.	-
00:45	*Then, I reduced both sides by 2. So I get* $4=\frac{8}{1+x}$.	*Vs-Sb*
01:03	*And then I wrote* $1+x=\frac{8}{4}$.	*Vs-Sb*
01.05	*Okay ma'am, the final answer is 1.*	

Figure 10. Subject S2's transcription data for well-posed problem solving

Figure 9 and Figure 10 denote that S2 transformed or modified a symbolic expression due to a new interpretation. S2 used verbal representation to formulates a symbolic expression. S2 was able to detect and correct the error of the final answer. S2 made translation between a verbal representation to symbolic representation to compute his solutions.

Based on Figure 3 and 7, the subject solution stages of solving ill-posed problem can be described as follows.

1. Subject used verbal representation to identify the problem.
2. Subject used translation between verbal representation and visual representation to describe the data.
3. Subject used translation between visual representation and symbolic representation to compute the answer.
4. Subject lacks to uncover necessary information about scale to justify her answers.

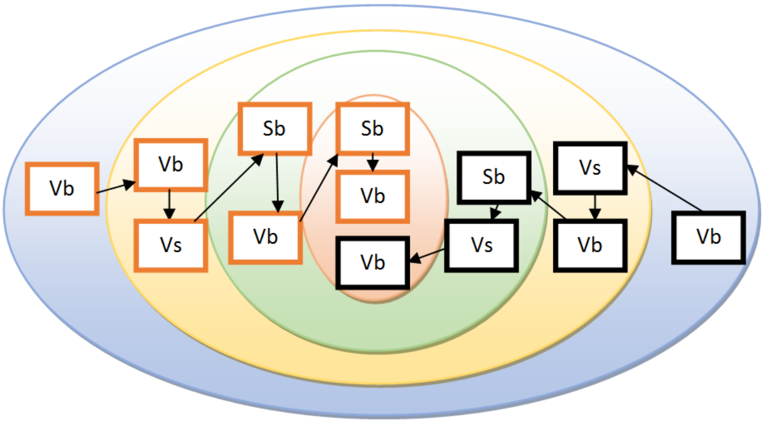

Figure 11. Stages of subject's mathematical representation in ill-posed problem-solving.

Information:

(light blue)	Identify the problem
(light purple)	Make a plan to solve the problem
(light orange)	Carry out the plan to solve the problem (to compute and justify the answer)
(light red)	Detect and correct an error of procedures and the answer
Subject 1 (red box)	Subject 2 (black box)
Vb: Verbal representation	Vs: Visual representation
Sb: Symbolic representation	→: translation

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Based on Figure 5 and 9, the subject solution stages of solving well-posed problem can be described as follows.

1. Subject used verbal representation to identify the problem.
2. Subject used translation between verbal representation and symbolic representation to describe the data.
3. Subject used translation between symbolic representation and verbal representation to compute the answer.
4. Subject used verbal representation to justify her answers.

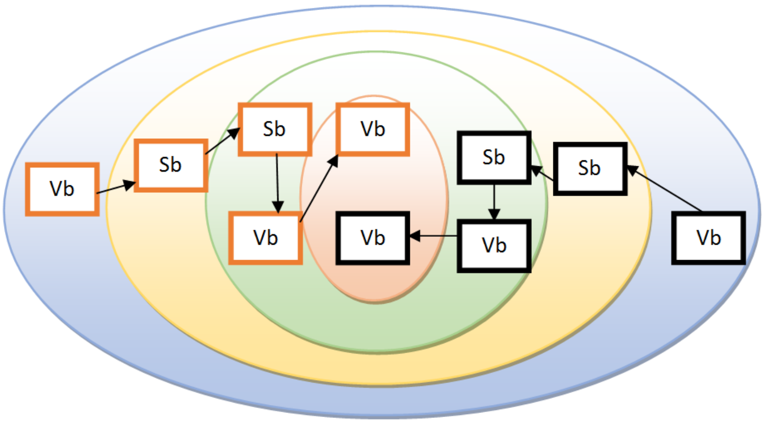

Figure 12. Stages of subject's mathematical representation in well-posed problem-solving

Information:

(blue)	Identify the problem
(purple)	Make a plan to solve the problem
(orange)	Carry out the plan to solve the problem (to compute and justify the answer)
(pink)	Detect and correct an error of procedures and the answer

Subject 1 (red box) Subject 2 (black box)

Vb: Verbal representation Vs: Visual representation
Sb: Symbolic representation →: translation

The findings of this study indicate that some participants are very creative in representing mathematical ideas in problem-solving, especially ill-posed problems. Students' creativity can be fostered through problem-solving (Grégoire, 2016; Shriki, 2010). The participants in the study showed a novelty solution. The novelty solution refers to answer that new for students (Rofiki, 2015). Even so, there are still many students who have difficulty in solving ill-posed problems. This is due to the complexity of ill-posed problems so that students fail to solve the ill-posed problem. Feltovich et al. (1996) asserted that the complexity of problems could inhibit students' success in problem-solving. Students can use reflective thinking maximally to defeat the complexity (Rofiki et al., 2017b). Therefore, the implication of this study in learning is that, educators should provide scaffolding to students in the form of question prompts and peer interactions so that students succeed in overcoming difficulties or obstacles to learning. Both question prompts and peer interactions are proper scaffolding in ill-structured problem-solving processes. Some studies suggested the utilization of scaffolding to overcome students' difficulties or mistakes as good impact in mathematics learning (Rofiki et al., 2017a; Roschelle et al., 2010; Shin & Song, 2016; Wischgoll et al., 2015).

4. CONCLUSION

The present study found that the two participants have different typologies in how they use representations in solving well-posed and ill-posed problems. This leads us to assert that there were well-defined typologies of solvers. Characterization of the solvers makes evident that there is a strong relationship between success in solving well-posed and ill-posed problems and skill in the translation of representations. Student's representation has to correct and strongly linked to each other so that they could manage information in problem-solving successfully (Dreyfus, 1991). Furthermore, the next study needs to explore the trigger factors of good mathematical representation in problem-solving. Moreover, the educator should provide various problems, especially ill-posed problems in mathematics learning, to improve students' representation and problem-solving ability. To enhance students' ill-posed problem-solving performance, scaffolding is necessary to develop students' thought processes or problem-solving skill. For further study, it is also crucial to examine the characteristics of scaffolding in students' ill-posed problem-solving.

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