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Analysis of students' mistakes in solving system of linear equation in three variables: A case on HOTS problems

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A B S T R A C T
One of the serious mathematics problem faced by students happen when it relates to Higher Order Thinking Skills (HOTS) type which is compulsary of 2013 Curriculum. The purpose of this research is to describe the analysis of these types of students' mistakes in solving mathematics problem of linear equation in three variables which used HOTS at SMAN Rambipuji Jember. The method used in this research is qualitative descriptive which describe the analysis' result on type of students mistakes based on Newman error indicator in terms of the students' cognitive level. The results showed that students with high cognitive level experiencing all kinds of errors with tendency to make a mistake on process error and writing the final answer. While students with moderate cognitive level is experiencing four types of errors, they are errors to understand the problem, transform problem, process error, and writing the final answer. Students with low cognitive level experience three types of errors, they are transformation problem errors, process errors, and error in writing final answer with tendency to experiencing these three types of errors in problem completion.

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1. INTRODUCTION

Evaluation is a very important activity in the education field to provide feedback of previous learning. Evaluation in education includes two steps, namely measurement and assessment (Leighton & Gierl, 2007). One of the tools to measure students' understanding is by giving them a task in the form of test and practice as a final assessment of learning. Sometimes a question becomes a problem for students to solve (Liljedahl, 2018). The problem is defined as the gap between the current situation and the purpose which the way to overcome these gaps are not immediately visible (Goldstein, 2011). One of the serious issue faced by students is higher order problem as the compulsory need of Indonesia curriculum (Agung & Schwartz, 2007). Students experienced some difficulties on solving mathematics word problem because their ability to understand the meaning of the problem (Auzar, 2017; Bernardo, 1999; Kristanti et al., 2018; Morin, Watson, Hester, & Raver, 2017). Those happened because they are not familiar in solving real life and complex problem which used Higher Order Thinking Skill (HOTS) to solve.

In nowdays era, higher order thinking is really needed to solve a lot of complex problem as the advancement of technology. Chidozie et al. (2014) said that students who were taught to think critically as higher order thinking showed a good influence in the development of their technological achievement. According to their statement, higher order thinking become very important and crucial needs in this high-advanced of technology era. An expert defined Higher Order Thinking Skills (HOTS) is a way of thinking that is higher than the memorized the facts, found the fact, or applied the rules, formulas, and procedures (Brookhart, 2010). He believed that HOTS includes the ability of critical thinking skills, logical reflective, metacognitive, and creativity. HOTS include critical thinking, creative thinking, problem solving, and decisions making (Ku et al., 2008). HOTS will develop if people deal with problems that are unknown, challenging question, or facing uncertainty. HOTS level include the students' ability or skills to analyze, evaluate, and create. However, most of students still do mistakes when the are required to use their HOTS ability. Therefore, identifying students mistakes can really benefit to fascilitate them to develop their higher order thinking skill.

There are several theories that usually used in analyzing student errors in solving problems. One of those is Newman Error Analysis Theory error (Prakitipong & Nakamura, 2006; Rohmah & Sutiarso, 2018; White, 2010). According to Newman, students' mistakes in solving math problems can be divided into five types of errors: (1) reading error; (2) comprehension error; (3) transformation error; (4) process error; (5) encoding error (Prakitipong & Nakamura, 2006; Rohmah & Sutiarso, 2018). Based on Newman Error Analysis we can obtain several indicators. (1) Reading error, students' mistakes classified in problem reading error, if students can not read the questions, read the questions incorrectly, can not write the key word, write the key word incorrectly, and can not mention the key word that exist on the question. (2) Comprehension errors (errors in understanding), students' mistakes classified in error to understand the problem if students can read the question but write on the form of symbols without any clear explanation, students can not write what is stated and what is asked on the question, or students write what is stated and asked on the question incorrectly. (3) Transformation error, students' mistakes classified in the transformation error if the student can not write example *(suppossion)*, students can make a supossion but can not explain it, students can not/incorrectly create mathematical models, and use a wrong method to solve problems. (4) Process error, students's mistakes classified in process error if students can not continue the solution procedure (stuck), students calculate wrongly due to any misuse of concepts and rules of mathematics, and students can not write and explain calculation phase precisely. (5) Encoding errors (error in writing the answers), students' mistakes classified in the final writing error if the student have solve the problems but did not write the conclusion, student write the conclusion but inappropriate to final calculation they got.

Meanwhile, students face some difficulties when they have to solve linear equation in three variables (Andrews & Sayers, 2012; Papaieronymou, 2007). This is happen when they have to solve real-life problem. Actually there are some cases in the real life related to linear equation in three variables, such as in arithmetic social. Some people sometimes buy three or more items in the store, but they do not see the real price of it, and finally they have to pay certain amount of money. This case an example of three linear equation in three variable case. In such kind of problem, students face difficulties to understand the problem and fail to find the final solution. Moreover, some researchers only described about how the error happened without any distingtion in the variables. Meanwhile, the error is always different based on students' understanding about certain concept. Such what Abdullah et al. (2015) research, which said that

Newman error criteria always happen for all level of students. Based on those facts, researchers want to analyze students' error in solving mathematics problem of linear equation in three variables. Through this research, researchers aimed to describe the analysis of the types of students' error in mathematics problem solving at linear equation in three variables which used Higher Order Thinking Skills (HOTS) by Newman Error Analysis in terms of the students' cognitive level.

2. METHOD

The descriptive qualitative research was conducted in SMA Rambipuji Jember. Researchers took six students as research subjects who have different cognitive levels consisting of two students who have a high cognitive level, the two students who have moderate cognitive level, and two students who have low cognitive level. Research's subject ellection was based on four criteria which is: (1) subject candidates have studied linear equation in three variables; (2) Subject candidates have done daily exam linear equation in three variables; (3) Mathematics teacher's information about good mathematics communication abbility to dig the information on interview become easier; (4) Students' agreement to be a research's subject.

The main instrument in this research is the researchers themselves, while the supporting instruments consist of three instruments as follows: (1) The question test linear equation in three variables of Higher Order Thinking Skills (HOTS) types which consist of three question with Newman Error Indicator. This test is used to determine the types of students' error in solving linear equation in three variables which used Higher Order Thinking Skills (HOTS); (2) Interview's guideline in this study is a list of questions that used to find out more about the types of students' error on solving problems linear equation in three variables which used Higher Order Thinking Skills (HOTS).

3. RESULTS AND DISCUSSION

Based on the research, there are several error that faced by student when they solved mathematics problem. Those are:

a. The error face by student at high level cognitive

   The test and interviewed shows that students who have high level cognitive performing all kinds of errors which are error in reading the problems, error in understand the problem, error in transforming the problem, error in processing skills, and error in writing the final answer. However, students at this level are more likely to make mistakes in processing skill and writing the final answer. Error in reading the problem occurs because students are less appropriate to write key words on the answer sheet. This because, first, students understand the known-problem but do not write it clearly. However, they write it into their own symbol without being given the right to write and students less is known of the matter. Second, the error occurred because the students misunderstood the sentences in question. Thirdly, error in transforming the matter, this occurs because students cannot/incorrectly changed into the form of a mathematical problem precisely. They face difficulty to change the sentences in question into a mathematical model. The fourth is error in processing skills, this error occurred because the students have not been true in the calculation. Furthermore, they misunderstood about concepts and rules of mathematics. Students are less scrupulous in writing mathematical symbols, resulting in one student in the calculation process. The last error was error in writing the final answer. This error occurs because

students write the wrong final calculation and the student one of the previous stage. The errors can be seen in the following description.

1. Reading error
   The student's error at this section can be seen in Figure 1.

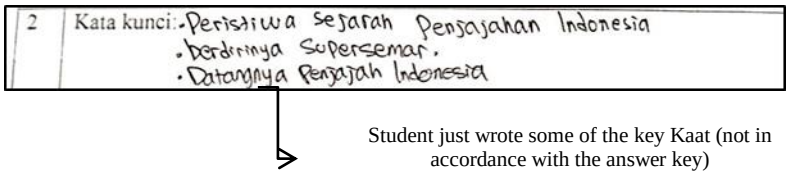

Figure 1. Reading error

   Figure 1 shows that student wrote down the keywords wrongly. This because there are some keywords which are not written like the longest, shortest side and the other side as in the answer key.

2. Comprehension error
   Student did the error which can be seen in the Figure 2.

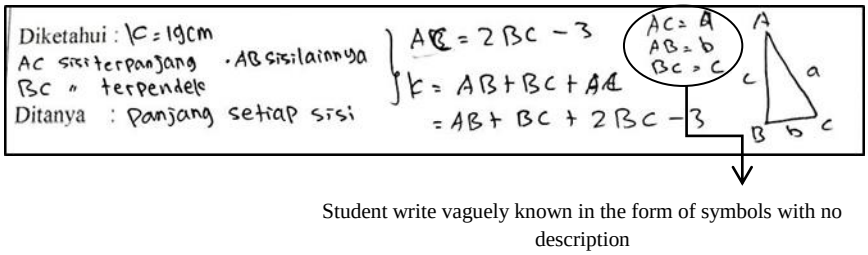

Figure 2. Comprehension error

   Figure 2 shows that student wrote the unknowninformation, but it is not clear. It happened because student write it into their own symbol but not given a clear explanation as AC = a, AB = b, and BA = c and students also write her assumption and mathematical models in the column is known.

3. Transformation error
   The transformation error can be seen in Figure 3.

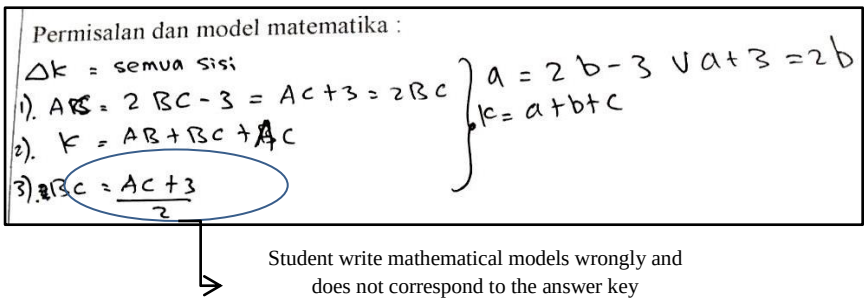

Figure 3. Transformation error

   Figure 3 shows that student did not write her assumption and wrongly wrote down a mathematical model (not in accordance with the answer key).

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4. Processing skill error
   Student's process skill error can be seen in Figure 4.

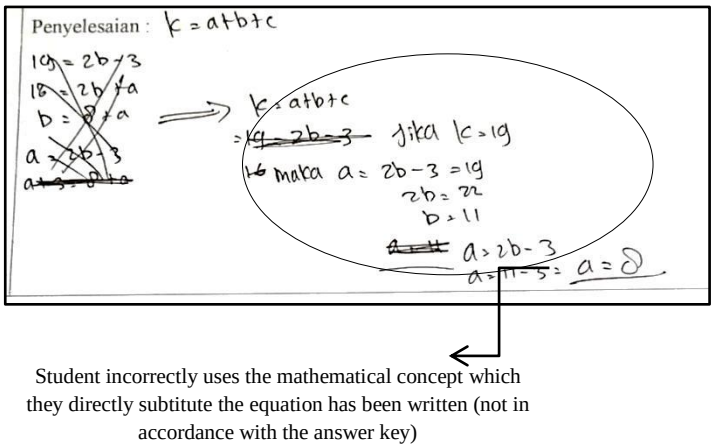

Figure 4. Processing skill error

   Figure 4 shows that student used mathematical concepts which directly subtitute the wrong mathematical model on other equations that have been made in the previous stage.

5. Encoding error
   Student's error at this stage can be seen in Figure 5.

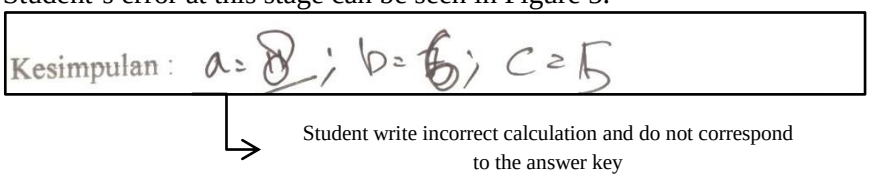

Figure 5. Encoding error

   Figure 5 shows that student wrote the incorrect calculation results, so that students have to write a final answer error.

b. The error of student with moderate cognitive level
   The test and interview transcript show that students with medium cognitive level is experiencing four types of errors which are errors to understand the problem, error to transform the problems, error to process information, and error in writing the final answer. Students with medium cognitive level were more likely fault to transform the matter, error in processing skills and error in writing final answer. First, the error occurred because students fail to understand the problem when they write what they understand: as well as the students write the note with their own symbol which is not given. Second, the transformation error problem. This error occurs because students cannot/incorrectly changed into the form of math problems correctly. Students are still difficult to change the sentences in question into a mathematical model. The third is the process skill error. This error occurred because the students incorrectly calculate the solution. The students incorrectly use the concept of substitution, elimination, and mix method. The fourth is error in writting the final answer. This case happened because students write the final answers based on the wrong calculation and the student did not write the final answer on the answer sheet. These errors can be seen in the following explanation.

1. Comprehension error
   Student's error in this phase can be seen in Figure 6.

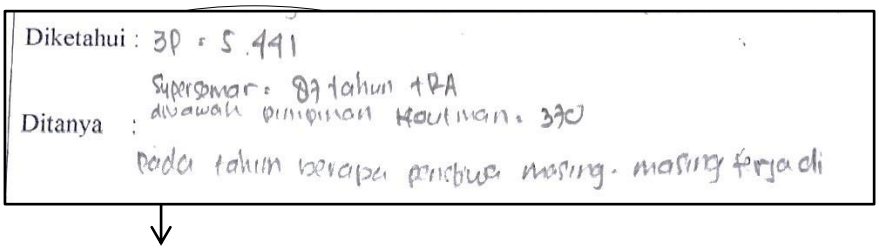

Figure 6. Comprehension error

   Student wrote the note and asked inappropriately (not in accordance with the answer key). Furthermore, student wrote the symbol without any obvious explanation.

2. Transformation error
   Student's error can be seen in Figure 7.

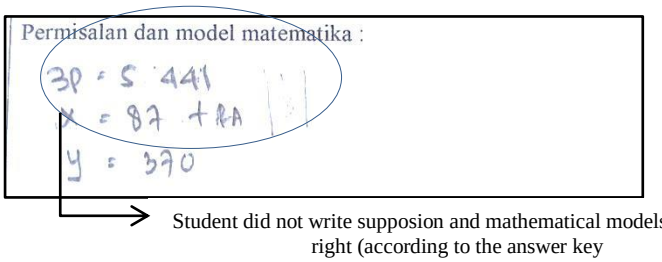

Figure 7. Transformation error

   Student cannot wrote an assumption and mathematical models correctly. Student wrote symbol P without any clear notes. Therefore, student experience a transformation error problem.

3. Processing skill error
   Student's error in this stage can be seen in Figure 8.

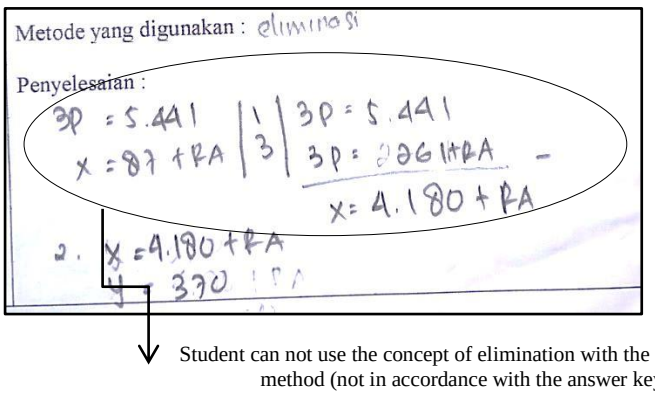

Figure 8. Process error

Student could not be resolved questions correctly. In this case, student incorrectly use the elimination method (not in accordance with the answer key).

4. Encoding error
   Student's error in this phase can be seen in Figure 9.

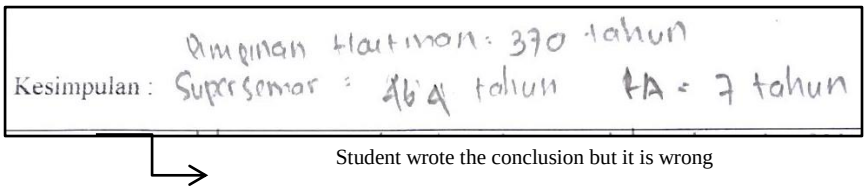

Figure 9. Encoding error

The student's work shows that student wrote wrong conclusion from the calculation in the preceding stage. Hence, student have to write a final answer error.

c. The error of student with low cognitive level
   It can be seen from the test and interview that students with low cognitive level experience three types of errors which are errors of transformation, error of processing, and error in writting the final answer. First, the problem of transformation errors. These errors occur because students cannot/incorrectly changed into the form of mathematics problems correctly. Furthermore, they still face difficulty to change the sentences in question into a mathematics model. Second, error in processing. This error occurs because the student has not been true in the calculation. They incorrectly use the concept of substitution, elimimnation, and mix method. The third was a mistake to write the final answer. This happened because students did not write the final answer on the answer sheet. Moreover, students are not able to solve problems at an earlier stage. For errors can be seen in the following figure:

   1. Transformation error
      Student's error can be seen in Figure 10.

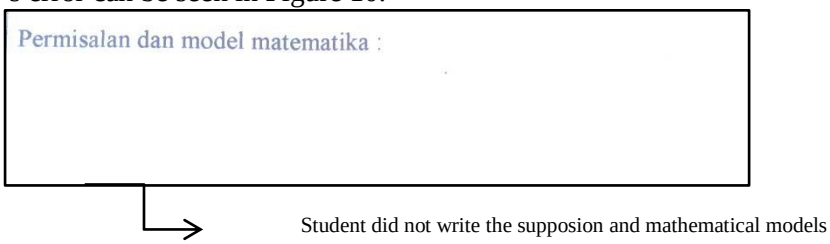

Figure 10. Transformation error

Figure 10 shows that student did not write any assumption and mathematical models, so that student experience a transformation error problem.

2. Process error
   Student's error in this phase can be seen in Figure 11.

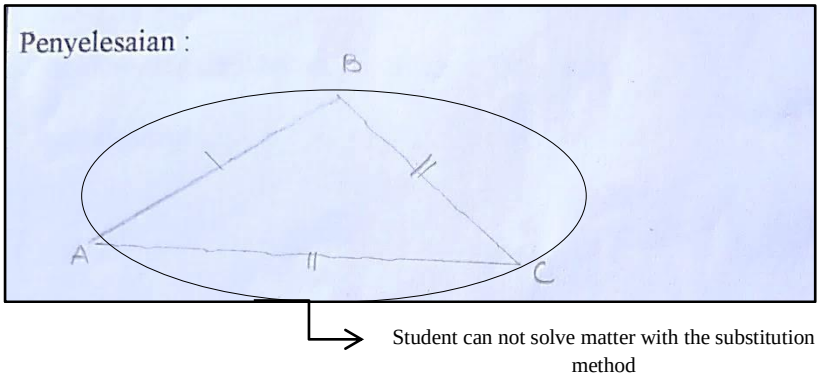

Figure 11. Process error

The student's work shows that student can not use substitution methods in solving problems and inappropriate to the answer key. This happened because student has not been able to make examples and convert problems into mathematical models in the previous stage.

3. Encoding error
   Student's error in this phase can be seen in Figure 12.

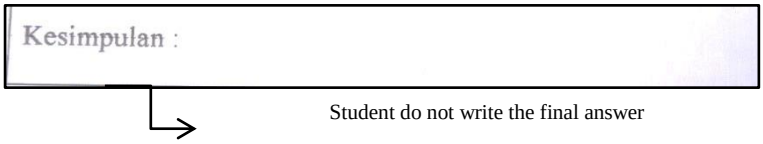

Figure 12. Encoding error

Figure 12 shows student's work in writing conclusion. It can be seen that student cannot write the final writing phase answer. It happened because in this level, student was fail to find the final solution. Therefore student found difficulties in writing the concution.

Mathematical mistakes made by the students are related to the cognitive activities, metacognitive ability, attitudes, and knowledge that they have (Afifah & Ningrum, 2018; Raduan, 2010). The difference in the levels of these attributes caused different mistakes committed and also the different capabilities each students have when solving mathematical problems. Based on those result, it can be seen that most of students in all level did same mistakes with what Newman said. Those errors consist of: (1) reading error; (2) comprehension errors; (3) transformation error; (4) Process error; (5) encoding error. This in line with the results of research conducted by (Rohmah & Sutiarso, 2018) that reported that students are more likely to make mistakes comprehension errors, transformation error, process error, and encoding error. Abdullah et al. (2015) also said that students at junior school levels do the same mistakes, consist of 5 types of error, in solving PISA-liked problem in the topic of fraction. However, both of the researches did not tell about the detailed error among cognitives level and there is no tendency for each error.

This research shows that the tendency happen for each levels, and it is bit different. High level students tend to do process error and encoding error (error in writing the final answer), this is truly supported by Wijaya et al. (2014) that said the high performing students made more mathematical processing errors. This happened because students are not careful

in the calculation process, resulting in errors in writing the final answer. While the medium level tends to make transformation errors, process errors and coding errors, this is because students have not been able to make examples and convert problems into mathematical models that will result in errors in the process and coding. In this research, students with low cognitive level experiencing, transformation error, process error, and encoding error, in other words they tend to do all type of mistakes. This partly different to Wijaya et al. (2014) that said low performing students made more comprehension errors and transformation errors than the high performing students. For the mathematical processing errors the opposite was found. Overall, actually there is notable differences of error between high performance students with the low performance students in the main activities of solving mathematics problems (Prakitipong & Nakamura, 2006).

4. CONCLUSION

This study found three characteristics of students' mistake in solving mathematics problem of linear equation in three variables. Firstly, students with high cognitive level experiencing all kinds of errors reading error, comprehension error, transformation error, process skill error, and encoding error with a major error at process error and encoding error (error in writing the final answer). Secondly, students with moderate cognitive level level experiencing comprehension error, transformation error, process error, and encoding error with a major error at transformation error, process skills errors and encoding error. Lastly, students with low cognitive level experiencing, transformation error, process error, and encoding error. They tend to do all type of mistakes. Based on the finding of this research, it is expected that teachers can consider the learning method that will be used, so that students with high, moderate, or low cognitive level can solve higer order thinking problem in mathematics well.

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