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# Dynamical Analysis of Fractional Order ZIKV Model with Nonlinear Incidence Rate

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**ABSTRACT**

The ZIKV model provided is derived by adapting the model proposed by recent research about basic fractional-order ZIKV model with a linear incidence rate. This study enhances the existing model by transforming it into a fractional-order ZIKV model with a nonlinear incidence rate.The use of nonlinear incidence rates in the ZIKV model is based on the understanding that ZIKV transmission does not always occur in proportion to the number of infected individuals. Adding detailed complexity to the ZIKV model could improve the representation of how ZIKV infection spreads between human populations and mosquito vectors. The model's equilibrium points are identified, and the stability conditions for each point are evaluated using the Routh-Hurwitz criterion. A numerical simulation using Predictor-corrector method is performed to validate the stability study results. Numerical simulations further demonstrate the impact of the order $\alpha$ on the stability of the equilibrium point inside the model.

**Keywords:** FDE; Equilibrium point; Fractional order; Stability analysis; Routh-Hurwitz



## 1 INTRODUCTION

ZIKV belongs to the family of pathogenic viruses called Flaviviridae, which also includes Japanese encephalitis, West Nile disease, Yellow fever, and Dengue fever. The mosquitoes are the primary vectors of ZIKV [4, 2, 5, 11]. Zika fever is brought on by ZIKV infection. It manifests as mild symptoms, including fever, red eyes, headache, joint discomfort, and a skin rash that appears two to seven days after incubation [6, 3]. ZIKV can spread from mother to fetus in the womb during pregnancy through blood transfusions, sexual contact, and bites from *Aedes aegypti* and *Aedes albopictus*.

Mathematical models play a crucial role in understanding the transmission dynamics of diseases, including the Zika Virus (ZIKV), within a specific area. In recent years, researchers

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have explored various mathematical epidemic models to capture the intricacies of ZIKV transmission. In this article, rather than ordinary differential equation, the ZIKV model is developed in fractional differential equation to considering memory effect. Ordinary differential equations have several limitations, such locally analyzed and do not consider the memory effect that appears in most biological models. Ordinary differential equations are also considered unable to provide information about the rate of change between two points. To overcome these limitations, fractional-order differential equations are used to complex phenomena modeling. The memory effect on fractional derivatives is written in the form of a definite integral, which means that the solution of the fractional derivative depends on all function values from the lower limit to the upper limit $t$ while the classical derivative only depends on the previous $t$. The memory effect on fractional derivatives makes fractional derivatives widely used by researchers to model the spread of diseases, especially diseases caused by virus infections. This is because virus infections are a natural event that involves the immune system. Early investigations of ZIKV fractional-order models focused mainly on linear incidence rates [2, 3, 4, 5, 6, 7, 8, 9, 10, 14, 17] or a general incidence rate [23]. These models typically accounted for transmission routes between humans and mosquitoes.

However, as our understanding of disease dynamics evolves, researchers are increasingly exploring more complex mathematical frameworks to represent the transmission dynamics of ZIKV better. In particular, there is a growing interest in mathematical models of ZIKV that incorporate fractional order derivatives [1, 14, 15, 16]. These models offer a more nuanced understanding of ZIKV transmission dynamics by considering the effects of memory on the nature of ZIKV transmission.

One such example of a ZIKV mathematical model with fractional order derivatives and nonlinear incidence rates is presented in [1].

$$\begin{aligned}
{}^{C}D_t^{\alpha_1} S_h &= \Lambda_h - \beta_h S_h (I_v + \delta I_h) - \mu_h S_h, \\
{}^{C}D_t^{\alpha_2} I_h &= \beta_h S_h (I_v + \delta I_h) - (\mu_h + \gamma + \eta_h) I_h, \\
{}^{C}D_t^{\alpha_3} R_h &= (\gamma + \eta_h) I_h - \mu_h R_h, \\
{}^{C}D_t^{\alpha_4} S_v &= \Lambda_v - \beta_v S_v I_h - \mu_v S_v, \\
{}^{C}D_t^{\alpha_5} I_v &= \beta_v S_v I_h - \mu_v I_v.
\end{aligned} \tag{1}$$

This model reflects the intricate interplay between human and mosquito populations and provides valuable insights into the dynamics of ZIKV transmission. By incorporating fractional order derivatives and nonlinear incidence rates, researchers aim to develop more accurate and comprehensive models that can inform public health interventions and strategies for controlling the spread of ZIKV.

In this article, the ZIKV model is developed by modifying model in [1]. The use of nonlinear incidence rates in the ZIKV model is based on the understanding that virus transmission does not always occur in proportion to the number of infected individuals [12, 24, 13]. In contrast, factors such as population density, level of immunity, and human and vector behavior can influence transmission patterns in complex and nonlinear ways. Adding detailed complexity to the ZIKV model could improve the representation of how ZIKV infection spreads between human populations and mosquito vectors. By describing the relationship between the number of individuals susceptible to infection, the level of contact between individuals, and other environmental factors in a nonlinear manner, this model can provide more accurate estimates of the development of the epidemic and the potential spread of ZIKV than the recent research [1].

In addition, nonlinear incidence rates also make it possible to model the effects of control interventions, namely the use of mosquito nets, insecticides, and hospital treatment. Thus,

the addition of nonlinear incidence rates in the ZIKV model opens up opportunities to launch various strategic interventions and design public policies that are more effective in dealing with the spread of this disease.

# 2 PRELIMINARIES

In the study of fractional calculus, the order of integrals and derivatives is denoted by $\alpha$ and represented as $D_t^\alpha = \frac{d^\alpha}{dt^\alpha}$. Generally, $\alpha$ can be any integer, fraction, or complex number $\alpha = p + iq$, where $p$ and $q$ are real numbers. Calculus Fractional is an advanced branch of calculus that studies fractional integrals and fractional derivatives. Fractional derivatives are believed to provide a more accurate description of actual systems when compared to conventional derivatives. The memory effect in fractional derivatives implies that the complete function value influences the solution of the derivative within the specified range. The subsequent passage is the precise definition of the Caputo fractional derivative, as stated by [19].

**Definition 1** *Let $\alpha \in \mathbb{R}^+$ and $n = \min\{z \in \mathbb{Z} : z \geq \alpha\}$. It is known that the Caputo fractional derivative of $y(t)$ in order $\alpha$, $n-1 < \alpha < n$ on $t > a$ is*

$$^C D_a^\alpha y(t) = \frac{1}{\Gamma(n-\alpha)} \int_a^t (t-\xi)^{n-\alpha-1} y^{(n)}(\xi) d\xi.$$

The symbol $\Gamma$ denotes the Gamma function. The fractional derivative, which is defined for $\alpha \in \mathbb{R}^+$, is a component of the fractional differential equation (FDE). [20]. The dynamical analysis of the fractional order model is carried out by determining the equilibrium point using Definition 2 and performing stability analysis of the equilibrium points using Theorem 1.

**Definition 2** *Equation*

$$^C D_t^\alpha y(t) = f(t, y(t))$$

*identified as Caputo FDE, the initial condition is detailed below,*

$$^C D_t^k y(0) = b_k \quad k = 1, 2, .., n-1. \tag{2}$$

The stability of the equilibrium point $y^* = (y_1^*, y_2^*, .., y_M^*)$ in the nonlinear system of FDEs may be ascertained by employing Theorem. 1 (See [21]).

**Theorem 1** *The equilibrium point $y^* = (y_1^*, y_2^*, .., y_M^*)$. The system becomes locally asymptotically stable for every eigenvalues $\lambda_i, \quad i = 1, 2, ...M$ of the Jacobi matrix $J = \frac{\partial f_i}{\partial y}$, assessed at the equilibrium point, fulfil the subsequent condition:*

$$|\arg(\lambda_i)| > \frac{\alpha\pi}{2}. \tag{3}$$

**Theorem 2** *The equation that describes the features of the Jacobian matrix at the equilibrium points is denoted by $P(\lambda) = \lambda^n + c_1\lambda^{n-1} + c_2\lambda^{n-2} + ... + c_n = 0$. The polynomial equation $P(\lambda)$ will possess n roots that comply with (3) if and only if the Routh-Hurwitz requirements for a fractional order system are satisfied [22].*

# 3 MAIN RESULT

The ZIKV model shown below is derived by adapting the model proposed in the reference [1]. Incorporating nonlinear incidence rates in the ZIKV model is predicated on recognizing that viral transmission does not consistently transpire in direct correlation to the number of infected humans. Nonlinear incidence rates enable modeling control strategies, such as mosquito nets, pesticides, and medical care.

$$\begin{aligned}
{}^{C}D_t^{\alpha_1} S_h &= \Lambda_h - \frac{\mu_4 \beta_h S_h}{1+\psi I_h}(I_v + \delta I_h) - \mu_h S_h, \\
{}^{C}D_t^{\alpha_2} I_h &= \frac{\mu_4 \beta_h S_h}{1+\psi I_h}(I_v + \delta I_h) - (\mu_h + \gamma_h) I_h, \\
{}^{C}D_t^{\alpha_3} R_h &= \gamma_h I_h - \mu_h R_h, \\
{}^{C}D_t^{\alpha_4} S_v &= \Lambda_v - \mu_4 \beta_v S_v I_h - \zeta_v S_v, \\
{}^{C}D_t^{\alpha_5} I_v &= \mu_4 \beta_v S_v I_h - \zeta_v I_v.
\end{aligned} \tag{4}$$

with

$$\begin{aligned}
\mu_4 &= 1 - \mu_1, \\
\zeta_v &= \mu_v + \eta_v \mu_3, \\
\gamma_h &= \gamma + \eta_h \mu_2.
\end{aligned}$$

$S_h$, $I_h$, $R_h$, $S_v$, and $I_v$ denote the quantities of susceptible human individuals, infected human individuals, recovered human individuals, susceptible mosquito individuals, and infected mosquito individuals, respectively. The variables $\Lambda_h$ and $\Lambda_v$ represent the rate at which susceptible humans and susceptible mosquitoes are recruited, whereas $\mu_h$ and $\mu_v$ represent the natural mortality rates for humans and mosquitoes, respectively. The variable $\beta_h$ represents the rate at which the infection is transmitted from humans to mosquitoes. The variable $\beta_v$ represents the rate at which the infection is transmitted from mosquitoes to humans. The variable $\eta_h$ represents the rate at which humans recover from the infection with treatment. The variable $\gamma$ represents the rate at which individuals recover from the infection. The control parameters are denoted as $\mu_1$, $\mu_2$, and $\mu_3$.

Considering the definition of equilibrium point in Definition 2 and the system (4), we have

$$S_h = \frac{\Lambda_v \left(\zeta_v \left(I_h \beta_v \mu_4 - \zeta_v\right)\left(\delta \mu_4 \beta_h I_h - \mu_h \left(1 + \psi I_h\right)\right) + \mu_4^2 \beta_h \beta_v \Lambda_v I_h\right)}{\zeta_v \left(1 + \psi I_h\right)\left(I_h \beta_v \mu_4 - \zeta_v\right)} \tag{5}$$

$$R_h = \frac{\gamma_h I_h}{\mu_h}, \tag{6}$$

$$S_v = \frac{\Lambda_v}{\mu_4 \beta_v I_h - \zeta_v}, \tag{7}$$

$$I_v = \frac{\mu_4 \beta_v \Lambda_v I_h}{\left(\mu_4 \beta_v I_h - \zeta_v\right)\zeta_v}, \tag{8}$$

and

$$I_h \left(\mathscr{A} I_h^4 + \mathscr{B} I_h^3 + \mathscr{C} I_h^2 + \mathscr{D} I_h + \mathscr{E} I_h\right) I_h = 0 \tag{9}$$

with

$$\begin{aligned}
\mathscr{A} &= \beta_v^2\psi^2\mu_4^2\zeta_v^2(\mu_h+\gamma_h)\\
\mathscr{B} &= -2\beta_v\psi^2\mu_4\zeta_v^3(\mu_h+\gamma_h)+\beta_v^2\mu_4^2\zeta_v^2\left(-\Lambda_v\delta^2\mu_4^2\beta_h^2+\Lambda_v\psi\delta\mu_4\mu_h\beta_h+2\psi(\mu_h+\gamma_h)\right)\\
\mathscr{C} &= -2\beta_v\mu_4\zeta_v^3\left(-\Lambda_v\delta^2\mu_4^2\beta_h^2+\Lambda_v\psi\delta\mu_4\mu_h\beta_h+2\psi(\mu_h+\gamma_h)\right)+\psi^2\zeta_v^4(\mu_h+\gamma_h)\\
&\quad+\beta_v^2\mu_4^2\zeta_v^2(\delta\Lambda_v\beta_h\mu_4\mu_h+\gamma_h+\mu_h)+\beta_v^2\Lambda_v^2\beta_h\mu_4^3\zeta_v(-2\delta\beta_h\mu_4+\psi\mu_h)\\
\mathscr{D} &= -2\beta_v\mu_4\zeta_v^3(\delta\Lambda_v\beta_h\mu_4\mu_h+\gamma_h+\mu_h)-\beta_v\Lambda_v^2\beta_h\mu_4^2\zeta_v^2(-2\delta\beta_h\mu_4+\psi\mu_h)\\
&\quad+\zeta_v^4\left(-\Lambda_v\delta^2\mu_4^2\beta_h^2+\Lambda_v\psi\delta\mu_4\mu_h\beta_h+2\psi(\mu_h+\gamma_h)\right)+\beta_v^2\Lambda_v^2\beta_h\mu_4^3\mu_h\zeta_v-\Lambda_v^3\beta_h^2\beta_v^2\mu_4^4\\
\mathscr{E} &= \zeta_v^4(\delta\Lambda_v\beta_h\mu_4\mu_h+\gamma_h+\mu_h)-\Lambda_v^2\zeta_v^2\beta_h\beta_v\mu_4^2\mu_h
\end{aligned}$$

Based on Equation (9), If $I_h=0$, the initial equilibrium point $E_0$ is obtained and referred to as the disease-free equilibrium (DFE).

$$E_0=\left(\frac{\Lambda_h}{\mu_h},0,0,\frac{\Lambda_v}{\zeta_v},0\right).$$

If $I_h\neq 0$, then $I_h$ satisfy the quadratic equation below.

$$\mathscr{A}I_h^4+\mathscr{B}I_h^3+\mathscr{C}I_h^2+\mathscr{D}I_h+\mathscr{E}I_h=0. \tag{10}$$

Equation (10) possesses a real and unique positive root if and only if $\mathscr{R}_0>1$, which

$$\mathscr{R}_0=\frac{\mu_4\left(\Lambda_h\beta_h\zeta_v^2\delta+\sqrt{\Lambda_h\beta_h\zeta_v^2(\Lambda_h\beta_h\zeta_v^2\delta^2+4\Lambda_v\beta_v\mu_h(\gamma_h+\mu_h))}\right)}{2\mu_h\zeta_v^2(\mu_h+\gamma_h)}.$$

We call the point $I_h^*$ the real and unique positive root of equation (10). Put point $I_h^*$ into equations (5), (6), (7), and (8) to determine the second equilibrium point, known as the endemic equilibrium (EE), expressed as $E_1=\left(S_h^*,I_h^*,R_h^*,S_v^*,I_v^*\right)$, with

$$\begin{aligned}
S_h^* &= \frac{\Lambda_v\left(\zeta_v\left(I_h^*\beta_v\mu_4-\zeta_v\right)\left(\delta\mu_4\beta_hI_h^*-\mu_h\left(1+\psi I_h^*\right)\right)+\mu_4^2\beta_h\beta_v\Lambda_vI_h^*\right)}{\zeta_v\left(1+\psi I_h^*\right)\left(I_h^*\beta_v\mu_4-\zeta_v\right)}\\
R_h^* &= \frac{\gamma_hI_h^*}{\mu_h},\\
S_v^* &= \frac{\Lambda_v}{\mu_4\beta_vI_h^*-\zeta_v},\\
I_v^* &= \frac{\mu_4\beta_v\Lambda_vI_h^*}{\left(\mu_4\beta_vI_h^*-\zeta_v\right)\zeta_v},
\end{aligned}$$

In order to assess the stability of the equilibrium point, we employ linearization using a Jacobi matrix at $\hat{E}=\left(\hat{S}_h,\hat{I}_h,\hat{R}_h,\hat{S}_v,\hat{I}_v\right)$, written as.

$$J=\begin{bmatrix}
-\frac{\mu_4\beta_h(I_h\delta+I_v)}{1+\psi I_h}-\mu_h & -\frac{\mu_4\beta_hS_h\delta}{1+\psi I_h}+\frac{\mu_4\beta_hS_h(I_h\delta+I_v)\psi}{(1+\psi I_h)^2} & 0 & 0 & -\frac{\mu_4\beta_hS_h}{1+\psi I_h}\\
\frac{\mu_4\beta_h(I_h\delta I_v)}{1+\psi I_h} & \frac{\mu_4\beta_hS_h\delta}{1+\psi I_h}-\frac{\mu_4\beta_h\psi S_h(I_h\delta+I_v)}{(1+\psi I_h)^2}-\mu_h-\gamma_h & 0 & 0 & \frac{\mu_4\beta_hS_h}{1+\psi I_h}\\
0 & \gamma_h & -\mu_h & 0 & 0\\
0 & -\mu_4\beta_vS_v & 0 & -\mu_4\beta_vI_h-\zeta_v & 0\\
0 & \mu_4\beta_vS_v & 0 & \mu_4\beta_vI_h & -\zeta_v
\end{bmatrix}. \tag{11}$$

Then (11) is evaluated at $E_0$

$$J_0 = \begin{bmatrix} -\mu_h & -A_1\delta & 0 & 0 & -A_1 \\ 0 & A_1\delta - \mu_h - \gamma_h & 0 & 0 & A_1 \\ 0 & \gamma_h & -\mu_h & 0 & 0 \\ 0 & -A_2 & 0 & -\zeta_v & 0 \\ 0 & A_2 & 0 & 0 & -\zeta_v \end{bmatrix}$$

obtained the following characteristic equation.

$$(\mu_h + \lambda)^2 (\zeta_v + \lambda) \left(\lambda^2 + P_1\lambda + P_2 = 0\right) = 0, \tag{12}$$

with

$$\begin{aligned} P_1 &= \zeta_v + \gamma_h + \mu_h - A_1\delta \\ P_2 &= \zeta_v(\gamma_h + \mu_h - A_1\delta) - A_1A_2. \end{aligned}$$

According to equation (12), there are 5 eigenvalues. Specifically, the eigenvalues are $\lambda_{1,2} = -\mu_h$ and $\lambda_3 = -\zeta_v$. Considering Theorem 1, it must be proven that $\lambda_i < 0$ for $i = 1,2,..,5$. First, it is obvious that $\lambda_j < 0$ for $j = 1,2,3$ and $\arg(\lambda_j) = \pi$, implying that $\left|\arg(\lambda_j)\right| > \frac{\alpha\pi}{2}$. The eigenvalues $\lambda_4$ and $\lambda_5$ are the solutions to the quadratic equation component. The solutions of the quadratic equation have negative real components if fulfill the condition

$$\frac{A_1\zeta_v\delta + \sqrt{A_1\zeta_v(A_1\zeta_v\delta^2 + 4A_2(\mu_h + \gamma_h))}}{2\zeta_v(\mu_h + \gamma_h)} < 1.$$

This condition is met when $\mathscr{R}_0 < 1$. The disease-free equilibrium point $E_0$ is locally asymptotically stable if and only if the basic reproduction number $\mathscr{R}_0$ is less than 1.

To assess the stability of $E_1$, we evaluate the Jacobian matrix (11) at $E_1$.

$$J_1 = \begin{bmatrix} -B_1B_2 - \mu_h & -B_6 + B_7 & 0 & 0 & -B_5 \\ B_1B_2 & B_6 - B_7 - \mu_h - \gamma_h & 0 & 0 & B_5 \\ 0 & \gamma_h & -\mu_h & 0 & 0 \\ 0 & -B_3 & 0 & -B_4 - \zeta_4 & 0 \\ 0 & B_3 & 0 & B_4 & -\zeta_v \end{bmatrix}$$

so that it is obtained the characteristic equation below.

$$(\lambda + \mu_v)(\lambda + \mu_h)\left(\lambda^3 + Q_1\lambda^2 + Q_2\lambda + Q_3\right) = 0, \tag{13}$$

with

$$\begin{aligned} Q_1 &= B_1B_2 + B_4 - B_6 + B_7 + \zeta_v + \gamma_h + 2\mu_h, \\ Q_2 &= \mu_h^2 + (B_1B_2 + 2B_4 - B_6 + B_7 + 2\zeta_v + \gamma_h)\mu_h + (B_1B_2 - B_6 + B_7 + \gamma_h)(\zeta_v + B_4) \\ &\quad + \gamma_h B_1B_2 - B_3B_5 \\ Q_3 &= (\zeta_v + B_4)\mu_h^2 + ((B_1B_2 - B_6 + B_7 + \gamma_h)(\zeta_v + B_4) - B_3B_5) + \gamma_h B_1B_2(\zeta_v + B_4) \end{aligned}$$

Based on equation (13) obtained $\lambda_1 = -\mu_h$ and $\lambda_2 = -\mu_v$. Its obvious that $\lambda_J < 0$, for $J = 1,2$ and $\arg(\lambda_J) = \pi$ so that it satisfy stability condition $|\arg(\lambda_J)| > \frac{\alpha\pi}{2}$. The roots of the cubic

equation are denoted as $\lambda_{3,4,5}$, and the discriminant of the equation is expressed as follows.

$$\begin{aligned}\Delta &= -\begin{vmatrix} 1 & Q_1 & Q_2 & Q_3 & 0 \\ 0 & 1 & Q_1 & Q_2 & Q_3 \\ 3 & 2Q_1 & Q_2 & 0 & 0 \\ 0 & 3 & 2Q_1 & Q_2 & 0 \\ 0 & 0 & 3 & 2Q_1 & Q_2 \end{vmatrix}, \\ &= 18Q_1Q_2Q_3 + (Q_1Q_2)^2 - 4Q_3Q_1^3 - 4Q_2^3 - 27Q_3^2.\end{aligned}$$

Considering Theorem 2, the equilibrium point $E_1$ is locally asymptotically stable if and only if one of the following conditions is met.

(i) If $\Delta > 0$ and fulfil the subsequent requirement,

1. $Q_1 > 0$,
2. $Q_3 > 0$, and
3. $Q_1Q_2 > Q_3$.

(ii) If $\Delta < 0$, $\alpha < \frac{2}{3}$, and fulfil the subsequent requirement,

1. $Q_1 \geq 0$,
2. $Q_2 \geq 0$, and
3. $Q_3 > 0$.

(iii) If $\Delta < 0$, $\alpha > \frac{2}{3}$, and fulfil the subsequent requirement,

1. $Q_1 < 0$ and
2. $Q_2 < 0$.

(iv) $\Delta < 0$ and fulfil the subsequent requirement,

1. $Q_1 < 0$,
2. $Q_2 < 0$, and
3. $Q_1Q_2 = Q_3$.

# 4 NUMERICAL SIMULATION

Numerical simulations are performed at a certain time span $t$ and step size $h$ and parameter values that meet the requirements of existence and uniqueness of the solution using the Predictor-Corrector method. Simulations performed using the Predictor-Corrector method which has been widely used in performing numerical analysis of fractional-order systems. The parameter values used are presented in Table 1 below.

Simulation 1 can be interpreted as a condition in which transmission from humans to mosquitoes ($\beta_h$) and the transmission rate from mosquitoes to humans ($\beta_v$) are very small, so that the population in the infected human compartment converges to 0 due to the recovery rate with treatment, the natural recovery rate, and the death rate are quite large compared to the increase in the population in this compartment due to transmission between susceptible humans

Table 1: Parameter value

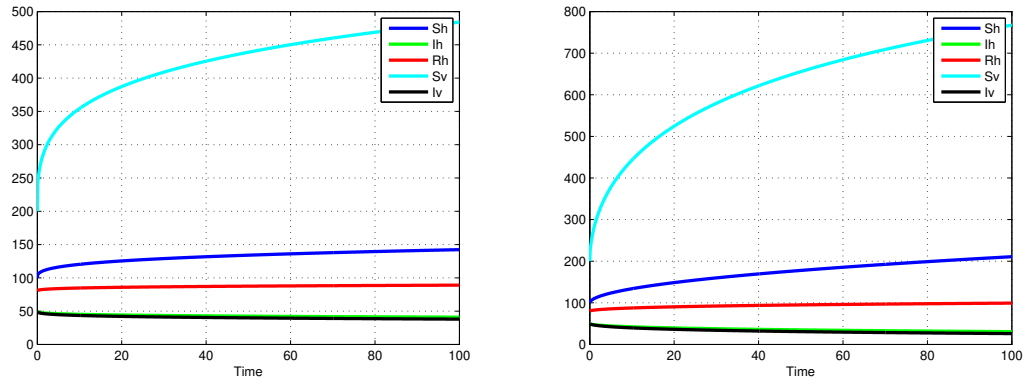
| Parameter | Paremeter value | Source |
|---|---|---|
| $\Lambda_h$ | 10 | [1] |
| $\Lambda_v$ | 100 | [1] |
| $\beta_h$ | $2 \times 10^{-7}$ | [1] |
| $\beta_v$ | $2 \times 10^{-8}$ | [1] |
| $\delta$ | 0.05 | [17] |
| $\mu_h$ | $\frac{1}{365 \times 60}$ | [2] |
| $\mu_v$ | $\frac{1}{14}$ | [2] |
| $\eta_h$ | 0.01 | [2] |
| $\eta_v$ | 0.001 | [2] |
| $\gamma$ | 0.05 | [24] |
| $\psi$ | 0.5 | assumed |

and infected humans and susceptible humans with infected mosquitoes. The population in the infected mosquito compartment also converges to 0 due to the higher mosquito mortality compared to the increase due to transmission between susceptible mosquitoes and infected humans. The population in the susceptible human and susceptible mosquito compartments continues to increase due to the large recruitment rate compared to the reduction in both compartments due to transmission and death. The simulation results confirm the findings of the stability analysis. The value of $E_0$ is not affected by the value of $\alpha$, but by the condition. The parameter $\alpha$ influences the rate at which the system approaches the desired value $E_0$. The system converges more rapidly to $E_0$ as the $\alpha$ value approaches 1. In the following simulation, the parameter values utilized are displayed in Table 2.

Table 2: Parameter value

| Parameter | Paremeter value | Source |
|---|---|---|
| $\Lambda_h$ | 10 | [1] |
| $\Lambda_v$ | 100 | [1] |
| $\beta_h$ | $2 \times 10^{-3}$ | [1] |
| $\beta_v$ | $2 \times 10^{-3}$ | [1] |
| $\delta$ | 0.05 | [17] |
| $\mu_h$ | $\frac{1}{365 \times 60}$ | [2] |
| $\mu_v$ | $\frac{1}{14}$ | [2] |
| $\eta_h$ | 0.01 | [2] |
| $\eta_v$ | 0.001 | [2] |
| $\gamma$ | 0.05 | [24] |
| $\psi$ | 0.5 | assumed |

The second simulation can be interpreted as a condition when the transmission rate from humans to mosquitoes ($\beta_h$) and the rate of transmission from mosquitoes to humans ($\beta_v$) are large enough. As a result, the number of populations in the infected human compartment converges towards a nonzero point because the reduction in the number of infected human populations with the rate of recovery with treatment, the rate of natural recovery, and the rate of death is not balanced with the addition of the number of populations due to transmission between susceptible humans and infected humans and susceptible humans with infected mosquitoes. The




(a) $\alpha = 0.3$ (b) $\alpha = 0.5$

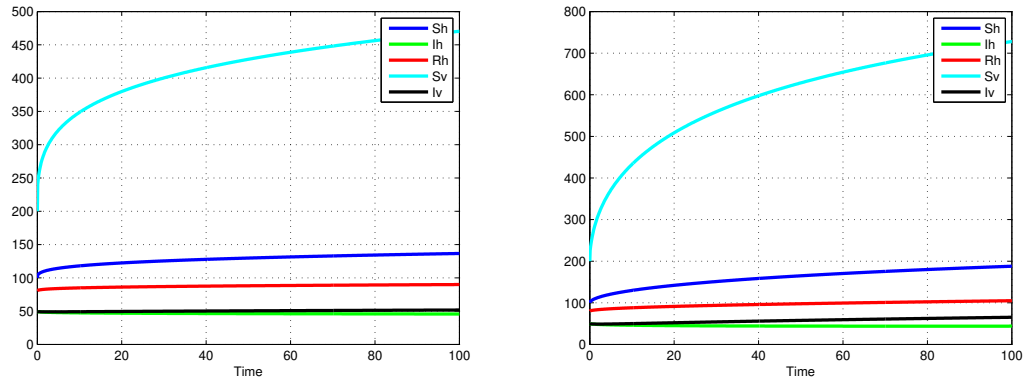
(c) $\alpha = 0.8$ (d) $\alpha = 1$

Figure 1: The population growth ($E_0$ asymptotically stable)

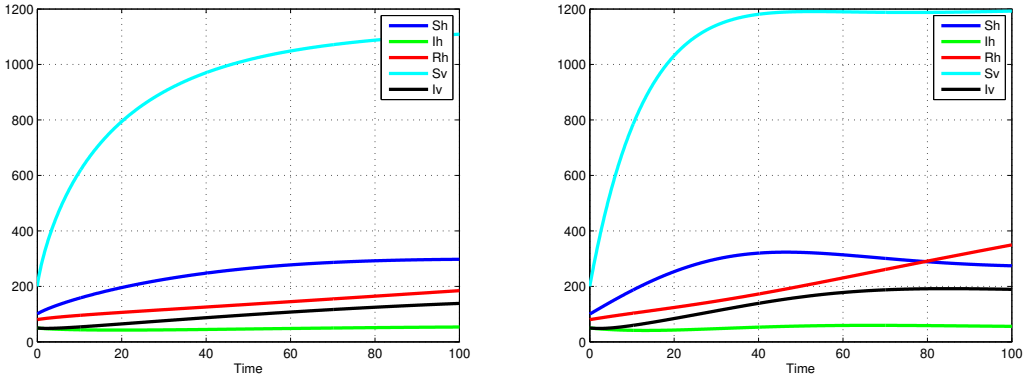
number of populations in the infected mosquito compartment also converges towards a nonzero point because the addition of the number of populations due to transmission between susceptible mosquitoes and infected humans is greater than the reduction due to natural death. In the second simulation, the stability criterion of $E_1$ is met, and this is corroborated by the simulation results shown in Figure 2. The stability of $E_1$ is not affected by the value of $\alpha$. The value of $\alpha$ influences the rate at which the system approaches convergence to $E_1$. The system converges more rapidly to $E_1$ as the $\alpha$ value approaches 1.

# 5 CONCLUSION

The ZIKV model in this paper consists of two populations divided into five compartments, and written as system of fractional differential equations. The model reveals two equilibrium points: Both equilibrium points demonstrate local asymptotic stability under different particular conditions. Numerical simulation supports the stability analysis results for each equilibrium point and illustrates the influence of $\alpha$ on their stability. The stability of $E_0$ is invariant to $\alpha$, however the stability of $E_1$ may be contingent upon the value of $\alpha$, depending on the particular conditions dictated by the chosen parameter values.




(a) $\alpha = 0.3$ (b) $\alpha = 0.5$




(c) $\alpha = 0.8$ (d) $\alpha = 1$

Figure 2: The population growth ($E_1$ asymptotically stable)

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