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Research Article

An Enhanced Vogel Approximation Method for Solving the Transportation Problem

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Abstract. The transportation problem is a classical linear programming model for allocating shipments from several supply points to several demand points at minimum total cost. A high-quality initial basic feasible solution is important because it can reduce the number of improvement iterations required by subsequent optimality tests. This study proposes an Enhanced Vogel Approximation Method (EVAM), a penalty-based modification of Vogel's Approximation Method that uses the three smallest active costs in each row or column. The method was tested on a balanced 5×5 transportation instance and compared with the Improved Vogel's Approximation Method (IVAM), followed by Stepping Stone improvement. The results show that IVAM produced an initial solution equal to the optimal cost of 59,356, whereas EVAM produced an initial cost of 60,727. After three Stepping Stone iterations, EVAM reached the same optimal cost of 59,356, with an initial optimality gap of 2.31%. These findings indicate that EVAM is simple and transparent, although it does not dominate IVAM on the tested instance.

Keywords: Transportation problem; Enhanced Vogel Approximation Method; initial basic feasible solution; Stepping Stone method; optimization.

1. Introduction

The transportation problem is one of the foundational formulations in operations research and linear programming. It describes the distribution of a homogeneous commodity from several sources to several destinations by considering supply capacity, demand requirements, and unit shipping costs. The formulation originates from Hitchcock's distribution model and Koopmans' economic allocation interpretation, and it later developed through linear programming and network algorithm approaches for obtaining minimum-cost solutions [1–4].

Solving a transportation problem usually starts with the construction of an initial basic feasible solution. The quality of this initial solution is important because it determines how far an improvement procedure must proceed before optimality is reached. Commonly used initial-solution methods include the North-West Corner Method, the Least Cost Method, and Vogel's Approximation Method (VAM). VAM is widely used because it employs an opportunity-loss penalty based on the two smallest active costs in each row or column; as a result, it often provides better initial solutions than rules based only on position or local minimum cost [5–10].

The development of initial-solution methods remains relevant because no single simple rule always produces the best solution for all cost structures. Several modifications have been proposed, including the Improved Vogel's Approximation Method (IVAM), the Total Opportunity Cost Matrix-Minimal Total approach, the Total Opportunity Cost Matrix-Supreme Cell approach, the two-highest-penalty method, the Bilqis-Chastine-Erma method, the Supply Selection Method, and other heuristic algorithms for balanced and unbalanced transportation problems [11–19]. These developments show that a simple, reproducible, and near-optimal initial-solution construction remains useful in transportation optimization.

The research gap addressed in this study is the need for a penalty rule that uses broader cost information than VAM without requiring a full opportunity-cost matrix transformation, as used by several Total Opportunity Cost-based methods. This study proposes the Enhanced Vogel Approximation Method (EVAM), a penalty rule that uses the three smallest active costs in each row or column. The inclusion of the third-smallest cost is intended to represent the local spread of costs more adequately, while the computational procedure remains simple because it does not construct an additional matrix.

The contribution of this study is fourfold. First, it presents an explicit mathematical formulation of the EVAM penalty. Second, it provides a reproducible allocation algorithm. Third, it compares EVAM with IVAM and Stepping Stone improvement on a balanced transportation instance. Fourth, it offers a proportional interpretation of the strengths and limitations of EVAM, so that the methodological claim does not exceed the available computational evidence.

2. Method

The method section describes the mathematical formulation of the transportation problem, the construction of the proposed Enhanced Vogel Approximation Method (EVAM), and the evaluation procedure used to assess the quality of the resulting initial basic feasible solution. The formulation is first presented to define the decision variables, objective function, and supply–demand constraints before the proposed

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allocation procedure is introduced.

2.1. Transportation Problem Formulation

This study considers a balanced transportation problem with m sources and n destinations. Let a_i denote the supply at source i, b_j denote the demand at destination j, c_{ij} denote the unit shipping cost from source i to destination j, and x_{ij} denote the amount allocated from source i to destination j. The balanced condition is expressed as

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j. \tag{1}$$

The cost-minimization transportation model is written as

$$\min Z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}, \tag{2}$$

subject to

$$\sum_{j=1}^{n} x_{ij} = a_i, \qquad i = 1, 2, \ldots, m, \tag{3}$$

$$\sum_{i=1}^{m} x_{ij} = b_j, \qquad j = 1, 2, \ldots, n, \tag{4}$$

$$x_{ij} \geq 0, \qquad i = 1, 2, \ldots, m,\ j = 1, 2, \ldots, n. \tag{5}$$

If total supply and total demand are unequal, a dummy row or column can be added before constructing the initial solution. In this article, the basic model, VAM, IVAM, and the Stepping Stone test are not placed in a separate preliminary section. The essential theoretical and operational material is instead incorporated into the Method section because it directly supports the computational procedure being tested [6, 7, 11].

2.2. Comparison Method and Optimality Test

VAM computes a penalty from the difference between the two smallest active costs in each row or column. If $C_{i(1)} \leq C_{i(2)} \leq \cdots$ denotes the ordered active costs in row i, then the VAM row penalty is

$$P_i^{\text{VAM}} = C_{i(2)} - C_{i(1)}. \tag{6}$$

The row or column with the largest penalty is selected, and the maximum possible allocation is assigned to the minimum-cost cell in the selected row or column. IVAM extends this idea by incorporating a total opportunity-cost structure to improve the quality of the initial solution in some cases [11, 12, 20].

The optimality of the solution is tested using the Stepping Stone method. This method evaluates each nonbasic cell through a closed path that passes through basic cells. For a minimization problem, the solution is optimal if all opportunity-cost changes satisfy $\Delta Z \geq 0$. If some $\Delta Z < 0$ remains, the allocation can be improved by shifting the allocation along the corresponding closed path [3, 4, 8, 9].

2.3. Research Design and Data

This study is computational-comparative. The EVAM rule is formulated mathematically, applied to a balanced 5×5 transportation instance, and then compared with IVAM. The supply and demand structure follows the numerical case used in the original manuscript and is adapted from the transportation problem example discussed by Korukoglu and Balli [11]. Total supply and total demand are both equal to 1,975 units; therefore, no dummy row or column is required.

Performance is evaluated using three measures: the initial-solution cost, the final cost after Stepping Stone improvement, and the percentage gap between the initial cost and the optimal cost. The percentage gap is computed as

$$\text{Gap}(\%) = \frac{Z_{\text{initial}} - Z_{\text{optimal}}}{Z_{\text{optimal}}} \times 100\%. \tag{7}$$

This measure is used to assess how close the initial solution is to the optimal solution.

2.4. Construction of the EVAM Penalty

Let R_i be an active row, and let $C_{i(1)} \leq C_{i(2)} \leq C_{i(3)} \leq \cdots$ denote the ordered active transportation costs in that row. Unlike VAM, which uses only the two smallest costs, EVAM uses the three smallest costs. The EVAM row penalty is defined as

$$P_i^{\text{EVAM}} = (C_{i(2)} - C_{i(1)}) + (C_{i(3)} - C_{i(1)}). \tag{8}$$

For an active column D_j, with ordered active costs $C_{j(1)} \leq C_{j(2)} \leq C_{j(3)} \leq \cdots$, the EVAM column penalty is

$$P_j^{\text{EVAM}} = (C_{j(2)} - C_{j(1)}) + (C_{j(3)} - C_{j(1)}). \tag{9}$$

The compact form of Eq. (8) and Eq. (9) is

$$P^{\text{EVAM}} = C_2 + C_3 - 2C_1. \tag{10}$$

The penalty increases when the smallest cost is much lower than the next two alternatives. Thus, EVAM retains the opportunity-loss interpretation of VAM while adding information about the local spread of costs.

If only two active costs remain, the EVAM penalty is reduced to $C_2 - C_1$. If only one active cost remains, the penalty is set to zero and the remaining allocation is made directly. When equal penalties occur, the row or column containing the smallest cost is selected. If a tie still occurs, the cell that allows the largest allocation is prioritized. This tie-breaking rule is specified to ensure that the computational process is reproducible.

2.5. EVAM Algorithm

The steps of EVAM are summarized in Algorithm 1.

The procedural advantage of EVAM is that it does not require the explicit construction of a Total Opportunity Cost matrix. In each iteration, the method only requires the three smallest active costs in each row and column, which makes the implementation relatively easy to carry out manually, in a spreadsheet, or through a computer program.

Algorithm 1 Enhanced Vogel Approximation Method

Require: Cost matrix $C = (c_{ij})$, supply vector a, demand vector b
Ensure: Initial allocation matrix $X = (x_{ij})$
Set $x_{ij} \leftarrow 0$ for all i, j.
while there is still active supply or demand **do**
  Compute P_i^{EVAM} for each active row using Eq. (8).
  Compute P_j^{EVAM} for each active column using Eq. (9).
  Select the active row or column with the largest penalty.
  Select the active cell (i, j) with the lowest cost in the selected row or column.
  Allocate $x_{ij} \leftarrow \min(a_i, b_j)$.
  Update $a_i \leftarrow a_i - x_{ij}$ and $b_j \leftarrow b_j - x_{ij}$.
  Delete row i if $a_i = 0$ and delete column j if $b_j = 0$.
end while
return X

3. Results and Discussion

The results and discussion section reports the computational outcomes obtained from the tested balanced transportation problem. The analysis focuses on the initial solution generated by EVAM, its comparison with IVAM, and the improvement obtained through the Stepping Stone optimality procedure.

3.1. Initial EVAM Solution

EVAM was applied to a balanced 5×5 transportation instance. In the first iteration, the largest penalty was obtained in destination column D_5, with a penalty value of 101. Repeated allocation based on Algorithm 1 produced the initial solution shown in Table 1. The initial total transportation cost was 60,727.

Table 1: EVAM allocation after Stepping Stone improvement

Source	D_1	D_2	D_3	D_4	D_5	Supply
S_1	0	0	461	0	0	461
S_2	277	0	0	0	0	277
S_3	1	0	0	116	239	356
S_4	0	0	0	0	488	488
S_5	0	60	0	0	333	393
Demand	278	60	461	116	1,060	1,975

Table 1 shows that the EVAM allocation satisfies all supply and demand constraints. However, the Stepping Stone test still found negative cost changes in several nonbasic cells. This means that the initial EVAM solution is feasible but not yet optimal.

3.2. Improvement Using the Stepping Stone Method

The initial EVAM solution was improved using the Stepping Stone method. After three improvement iterations, the allocation in Table 2 was obtained. The total cost decreased from 60,727 to 59,356.

The cost reduction of 1,371 indicates that an improvement procedure was still needed after constructing the initial solution. Based on Eq. (7), the initial EVAM cost gap relative to

Table 2: EVAM allocation after Stepping Stone improvement

Source	D_1	D_2	D_3	D_4	D_5	Supply
S_1	0	0	461	0	0	461
S_2	277	0	0	0	0	277
S_3	1	0	0	116	239	356
S_4	0	0	0	0	488	488
S_5	0	60	0	0	333	393
Demand	278	60	461	116	1,060	1,975

the optimal cost is

$$\frac{60{,}727 - 59{,}356}{59{,}356} \times 100\% = 2.31\%. \tag{11}$$

This result confirms that EVAM produces a feasible and near-optimal initial solution, but it does not directly reach optimality on the tested instance.

3.3. Comparison Between EVAM and IVAM

The comparison between EVAM and IVAM is presented in Table 3. IVAM produced an initial cost of 59,356, which was already optimal. EVAM produced a higher initial cost, but it reached the same optimal cost after Stepping Stone improvement.

Table 3: Comparison between IVAM and EVAM on the numerical instance

Method	Initial cost	Final cost	Gap (%)	Note
IVAM	59,356	59,356	0.00	Initial solution already optimal
EVAM	60,727	59,356	2.31	Three Stepping Stone iterations required

The cost comparison is visualized in Figure 1. The figure shows that the initial EVAM cost is above the IVAM cost, whereas the EVAM cost after improvement equals the optimal IVAM cost.

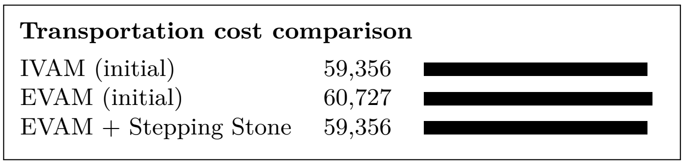

Fig. 1: Transportation cost comparison for IVAM, EVAM, and EVAM after Stepping Stone improvement.

The results in Table 3 and Figure 1 show that EVAM is not superior to IVAM in terms of initial-solution quality for this case. The advantage of EVAM is better interpreted as procedural simplicity because it does not require the construction of a total opportunity-cost matrix. This interpretation is consistent with recent literature showing that the performance of heuristic initial-solution methods depends strongly on the cost structure, problem size, and tie-breaking rules used [15, 16, 21, 22].

3.4. Implications and Limitations

Practically, EVAM can be used as an alternative initial heuristic when users need a concise procedure before conducting an optimality test. Conceptually, the use of the three smallest costs provides more local information than VAM, but this information is not necessarily sufficient to outperform methods that explicitly use a total opportunity-cost structure. Therefore, EVAM should be positioned as a simple and transparent initial method rather than as a universally better replacement for IVAM.

This study has two main limitations. First, the numerical evaluation uses only one balanced 5×5 transportation instance, so it cannot support a general claim of EVAM superiority. Second, the comparison is based only on total cost and the number of Stepping Stone iterations; it does not yet include large-scale computational experiments on random instances, real cases, unbalanced problems, or degenerate cases. Future research should compare EVAM with VAM, IVAM, TOCM-MT, BCE, SSM, the Maximum Range Method, and multipartite-graph approaches under various cost distributions and problem sizes [11, 12, 15, 16, 21, 22].

4. Conclusion

This study proposes EVAM as a three-cost penalty heuristic for constructing an initial basic feasible solution to the transportation problem. The EVAM penalty sums the differences between the smallest active cost and the next two active costs in each row or column. In the balanced 5×5 transportation instance, EVAM produced an initial solution with a cost of 60,727. After three Stepping Stone iterations, the solution reached the optimal cost of 59,356. IVAM produced an initial solution that was already optimal on the same case.

The main contribution of EVAM does not lie in a lower initial cost for the tested instance, but in a simple penalty construction that does not require forming a Total Opportunity Cost matrix. The method can be used as a transparent and reproducible initial heuristic, especially when paired with an optimality-improvement procedure. Further studies should test EVAM on a wider set of cases, analyze the effect of tie-breaking rules, handle degeneracy explicitly, and conduct computational comparisons against other modern initial-solution methods.

CRediT Authorship Contribution Statement

Shofiyatul Malik Mumtaza: Conceptualization, Methodology, Formal Analysis, Data Curation, Visualization, Writing - Original Draft. **Juhari:** Supervision, Validation, Writing - Review and Editing.

Declaration of the Use of AI or AI-Assisted Technologies

The authors declare that the original research manuscript did not use generative AI technologies for data analysis, result interpretation, or scientific conclusion drawing. AI-assisted technology was used only for language polishing, formatting refinement, and adaptation of the manuscript to the journal template.

Conflict of Interest Declaration

The authors declare that there are no financial or personal conflicts of interest that could influence the results or interpretation of this study.

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Data Availability

The numerical data supporting the findings of this study were adapted from the transportation problem case discussed by Korukoglu and Balli [11]. Details of the calculations and allocation steps are available from the corresponding author upon request.

References

[1] F. L. Hitchcock. "The Distribution of a Product from Several Sources to Numerous Localities". In: *Journal of Mathematics and Physics* 20.1-4 (1941), pp. 224–230. DOI: 10.1002/sapm1941201224. URL: https://doi.org/10.1002/sapm1941201224.

[2] T. C. Koopmans. "Optimum Utilization of the Transportation System". In: *Econometrica* 17 (1949), pp. 136–146. DOI: 10.2307/1907301. URL: https://doi.org/10.2307/1907301.

[3] A. Charnes and W. W. Cooper. "The Stepping Stone Method of Explaining Linear Programming Calculations in Transportation Problems". In: *Management Science* 1.1 (1954), pp. 49–69. DOI: 10.1287/mnsc.1.1.49. URL: https://doi.org/10.1287/mnsc.1.1.49.

[4] L. R. Ford and D. R. Fulkerson. "Solving the Transportation Problem". In: *Management Science* 3.1 (1956), pp. 24–32. DOI: 10.1287/mnsc.3.1.24. URL: https://doi.org/10.1287/mnsc.3.1.24.

[5] H. H. Shore. "The Transportation Problem and the Vogel Approximation Method". In: *Decision Sciences* 1.3-4 (1970), pp. 441–457. DOI: 10.1111/j.1540-5915.1970.tb00792.x. URL: https://doi.org/10.1111/j.1540-5915.1970.tb00792.x.

[6] S. K. Goyal. "Improving VAM for Unbalanced Transportation Problems". In: *Journal of the Operational Research Society* 35.12 (1984), pp. 1113–1114. DOI: 10.1057/jors.1984.217. URL: https://doi.org/10.1057/jors.1984.217.

[7] N. Balakrishnan. "Modified Vogel's Approximation Method for the Unbalanced Transportation Problem". In: *Applied Mathematics Letters* 3.2 (1990), pp. 9–11. DOI: 10.1016/0893-9659(90)90003-T. URL: https://doi.org/10.1016/0893-9659(90)90003-T.

[8] S. I. Gass. "On Solving the Transportation Problem". In: *Journal of the Operational Research Society* 41.4 (1990), pp. 291–297. DOI: 10.1057/jors.1990.50. URL: https://doi.org/10.1057/jors.1990.50.

[9] H. Arsham. "Postoptimality Analyses of the Transportation Problem". In: *Journal of the Operational Research Society* 43.2 (1992), pp. 121–139. DOI: 10.1057/jors.1992.18. URL: https://doi.org/10.1057/jors.1992.18.

[10] M. Mathirajan and B. Meenakshi. "Experimental Analysis of Some Variants of Vogel's Approximation Method". In: *Asia-Pacific Journal of Operational Research* 21.4 (2004), pp. 447–462. DOI: 10.1142/S0217595904000333. URL: https://doi.org/10.1142/S0217595904000333.

[11] S. Korukoglu and S. Balli. "An Improved Vogel's Approximation Method for the Transportation Problem". In: *Mathematical and Computational Applications* 16.2 (2011), pp. 370–381. DOI: 10.3390/mca16020370. URL: https://doi.org/10.3390/mca16020370.

[12] B. Amaliah, C. Fatichah, and E. Suryani. "Total Opportunity Cost Matrix-Minimal Total: A New Approach to Determine Initial Basic Feasible Solution of a Transportation Problem". In: *Egyptian Informatics Journal* 20.2 (2019), pp. 131–141. DOI: 10.1016/j.eij.2019.01.002. URL: https://doi.org/10.1016/j.eij.2019.01.002.

[13] B. Amaliah, C. Fatichah, E. Suryani, and A. Muswar. "Total Opportunity Cost Matrix-Supreme Cell: A New Method to Obtain Initial Basic Feasible Solution of Transportation Problems". In: *Proceedings of the 8th International Conference on Computer and Communications Management*. Association for Computing Machinery, 2020, pp. 151–156. DOI: 10.1145/3411174.3411198. URL: https://doi.org/10.1145/3411174.3411198.

[14] B. Amaliah, C. Fatichah, and E. Suryani. "Two Highest Penalties: A Modified Vogel's Approximation Method to Find Initial Basic Feasible Solution of Transportation Problem". In: *2021 13th International Conference on Information & Communication Technology and System (ICTS)*. IEEE, 2021, pp. 318–323. DOI: 10.1109/ICTS52701.2021.9608005. URL: https://doi.org/10.1109/ICTS52701.2021.9608005.

[15] B. Amaliah, C. Fatichah, and E. Suryani. "A New Heuristic Method of Finding the Initial Basic Feasible Solution to Solve the Transportation Problem". In: *Journal of King Saud University - Computer and Information Sciences* 34.5 (2022), pp. 2298–2307. DOI: 10.1016/j.jksuci.2020.07.007. URL: https://doi.org/10.1016/j.jksuci.2020.07.007.

[16] B. Amaliah, C. Fatichah, and E. Suryani. "A Supply Selection Method for Better Feasible Solution of Balanced Transportation Problem". In: *Expert Systems with Applications* 203 (2022), p. 117399. DOI: 10.1016/j.eswa.2022.117399. URL: https://doi.org/10.1016/j.eswa.2022.117399.

[17] E. M. D. B. Ekanayake and E. M. U. S. B. Ekanayake. "A Novel Approach Algorithm for Determining the Initial Basic Feasible Solution for Transportation Problems". In: *Indonesian Journal of Innovation and Applied Sciences* 2.3 (2022), pp. 234–246. DOI: 10.47540/ijias.v2i3.529. URL: https://doi.org/10.47540/ijias.v2i3.529.

[18] H. A. Hussein, M. A. K. Shiker, and M. S. M. Zabiba. "A New Revised Efficient of VAM to Find the Initial Solution for the Transportation Problem". In: *Journal of Physics: Conference Series* 1591.1 (2020), p. 012032. DOI: 10.1088/1742-6596/1591/1/012032. URL: https://doi.org/10.1088/1742-6596/1591/1/012032.

[19] K. Karagul and Y. Sahin. "A Novel Approximation Method to Obtain Initial Basic Feasible Solution of Transportation Problem". In: *Journal of King Saud University - Engineering Sciences* 32.3 (2020), pp. 211–218. DOI: 10.1016/j.jksues.2019.03.003. URL: https://doi.org/10.1016/j.jksues.2019.03.003.

[20] J. Nahar, E. Rusyaman, and S. D. V. E. Putri. "Application of Improved Vogel's Approximation Method in Minimization of Rice Distribution Costs of Perum BULOG". In: *IOP Conference Series: Materials Science and Engineering* 332.1 (2018), p. 012027. DOI: 10.1088/1757-899X/332/1/012027. URL: https://doi.org/10.1088/1757-899X/332/1/012027.

[21] F. A. Wireko, I. D. K. Mensah, E. N. A. Aborhey, S. A. Appiah, C. Sebil, and J. Ackora-Prah. "The Maximum Range Method for Finding Initial Basic Feasible Solution for Transportation Problems". In: *Results in Control and Optimization* 19 (2025), p. 100551. DOI: 10.1016/j.rico.2025.100551. URL: https://doi.org/10.1016/j.rico.2025.100551.

[22] N. Kalaivani and E. M. Visalakshidevi. "A Generalized Novel Approach to Transportation Problem Using Multi-Partite Graph Method". In: *Measurement: Sensors* 33 (2024), p. 101060. DOI: 10.1016/j.measurementsensors.2024.101060. URL: https://doi.org/10.1016/j.measurementsensors.2024.101060.