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Research Article

Comparative Analysis of Pension Fund Investment Strategies: Time versus Contribution Size in Logistic Growth Model

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Abstract. Pension fund planning requires strategies that balance contribution size, rate of return, and investment horizon to ensure long-term financial sustainability. This study compares early investing with small sustained contributions against late investing with large short-term contributions, using a logistic growth model $\frac{dA}{dt} = r_m A\left(1 - \frac{A}{K}\right) + C$ solved numerically via the 4th-order Runge-Kutta method. Seven scenarios across three categories (a core comparison, a robustness check, and a trade-off analysis) were designed with total contributions held equal at Rp360,000,000 to isolate the effect of investment timing. In the core comparison, early investing at 6% over 30 years yields Rp1,009,377,036 (total return multiplier 2.80x) versus Rp583,741,686 (1.62x) for late investing over 15 years, a 73% difference despite identical capital outlay. More critically, in the trade-off analysis, an early strategy at 5% over 30 years still outperformed a very late strategy at 8% over 10 years (Rp835,280,623 versus Rp551,414,017), confirming that even a three-percentage-point return advantage cannot compensate for a shorter horizon. These findings, consistent across different return rates and validated against the classical exponential-annuity benchmark, establish investment horizon as the dominant variable in long-term accumulation and offer practical guidance for early pension fund decision-making.

Keywords: Investment Strategy; Logistic Growth; Pension Fund; Runge-Kutta; Time Value of Money

1. Introduction

Pension fund planning is a fundamental component of long-term financial planning. During retirement, individuals generally no longer have active income, requiring optimal investment strategies to ensure financial sustainability [1]. In this context, investment decisions not only depend on contribution amounts but also on investment duration and rates of return [2].

In practice, there are two main strategies commonly compared in pension fund planning. The first strategy is starting investments early in life with relatively small but consistently made contributions over a long period (early investing). The second strategy is delaying investment until a certain age, but with larger contributions in a shorter period (late investing) [3].

As a concrete illustration, suppose individual A starts investing at age 25 until age 55 (30-year period) with fixed monthly contributions. Conversely, individual B only starts investing at age 40 until age 55 (15-year period) in the same investment instrument with identical rates of return, but with monthly contributions twice as large as individual A. If the total contributions of both individuals are equalized, then financially both strategies appear equivalent. However, contribution equality does not directly guarantee investment outcome equality.

This comparison design reflects common financial behavior realities: young individuals tend to have smaller monthly contribution capacity but longer investment horizons, while older individuals generally have greater financial capacity but shorter investment horizons [4]. Equalizing total contributions is done explicitly to isolate the influence of time as an independent variable, not to claim financial capacity equivalence between individuals, so that the difference in final values obtained can be fully attributed to investment duration differences.

The difference in outcomes between these two strategies is mainly influenced by the length of the accumulation period and the investment growth process over time. Investments started earlier have greater opportunity to experience significant value accumulation because the growth process lasts longer, even with smaller contributions. Conversely, investments started later depend on larger contributions to catch up on lost time, but face duration limitations for growth.

This problem shows the non-linear relationship between investment timing, contribution size, and final investment outcomes. Although this concept is often intuitively understood, quantitative studies that systematically compare various combinations of start timing, contributions, and returns are still limited, especially considering growth dynamics that are not purely exponential.

Several previous studies have examined long-term investment accumulation through time value of money approaches and compound interest mechanisms, which are generally modeled as exponential growth [5]. Additionally, periodic contribution models such as annuities are widely used to represent investments with regular deposits in pension fund planning [2]. These studies consistently show that timing is crucial, with early investing strategies tending to yield greater fund accumulation than late investing, even with smaller contributions [6], [7], [8].

However, most research still uses assumptions of linear or exponential growth without considering system limitations and non-linear dynamics in the investment accumulation pro-

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cess [8], [9]. Furthermore, investment strategy comparison studies are generally limited to two extreme conditions, early and late investing, without exploring broader scenarios [10], [11]. On the other hand, the use of logistic growth models that can capture growth slowdown due to certain limitations is still rarely applied in the context of investment with periodic contributions [12], [13]. Moreover, the relationship between mathematical models and real-world investment instruments that allow reinvestment mechanisms, such as fixed-income mutual funds based on sharia principles, has not been comprehensively discussed in the literature [14], [15].

Therefore, this research proposes a modeling approach using a logistic growth model capable of representing investment accumulation dynamics more realistically, especially in the long term. This model allows analysis of the interaction between periodic contributions and investment growth that experiences slowdown as accumulated value increases.

In modern investment practice, various instruments are available that enable long-term investment strategies with periodic contributions and growth-based reinvestment mechanisms [16]. One relevant instrument in this context is fixed-income mutual funds, including those based on sharia principles, which allow investment with relatively small nominal amounts and support value accumulation through automatic reinvestment [17]. These characteristics make such instruments aligned with the growth model assumptions used, so the analysis results obtained are not only theoretical but also have practical relevance in real-world implementation.

Against this background, this study asks to what extent the timing of starting investments determines pension fund accumulation value, and whether increases in contribution size or rates of return can fully compensate for a delayed start. Answering this question requires isolating time as an independent variable, which is a task complicated by the fact that early and late investors typically differ not only in their investment horizon but also in their monthly contribution capacity. This study addresses that challenge by equalizing total contributions across all scenarios, so that any observed differences in final accumulation value can be attributed solely to differences in investment duration.

To this end, the study implements a logistic growth model solved numerically via the 4th-order Runge-Kutta method, simulating seven scenarios across three structured categories: a core comparison of early versus late strategies at equal returns, a robustness check at higher return rates, and a trade-off analysis that directly tests whether sufficiently high returns can overcome the disadvantage of a shorter horizon. Together, these scenarios enable a systematic quantitative examination of how investment duration, contribution size, and rate of return interact in determining long-term pension fund outcomes.

Beyond its empirical findings, this study contributes a quantitative framework for analyzing pension fund accumulation under non-linear growth dynamics, which is an approach that has received limited attention in prior literature, particularly in the Indonesian context. For practitioners and young investors, the results offer evidence-based guidance on a decision that is both common and consequential: when to begin contributing to a pension fund, and whether waiting for a higher salary or better returns justifies the delay.

2. Literature Review

This section reviews the main concepts and previous studies related to long-term investment growth, pension fund planning, and the use of logistic models in financial analysis.

2.1. Fundamental Concepts of Long-Term Investment

In financial theory, long-term investment growth is generally explained through the concept of compound interest, where returns from an investment are reinvested to generate exponential growth [5].

Mathematically, the exponential growth model is expressed as:

$$A(t) = A_0 \left(1 + \frac{r}{n}\right)^{nt}$$

where $A(t)$ is the investment value at time t, A_0 is the initial investment, r is the annual rate of return, and n is the capitalization frequency per year.

This model is widely used due to its simplicity in representing the compounding mechanism. However, this formulation is based on several ideal assumptions: investment is made only once at the beginning of the period without additional contributions; growth rate is considered constant throughout; there are no external limitations affecting investment growth.

In practice, long-term investments, especially in pension fund planning contexts, generally involve periodic contributions and individual financial capacity limitations. Therefore, the simple exponential model becomes inadequate to realistically represent fund accumulation dynamics, requiring a more flexible modeling approach.

2.2. Logistic Growth Models in Financial Contexts

Logistic growth models were initially developed to model population dynamics [18], but their structural properties make them equally relevant for modeling financial investment accumulation. In the early phase of investment, when the accumulated value remains small relative to the carrying capacity, growth proceeds rapidly in a manner analogous to exponential compounding. As the investment value increases, however, growth begins to slow, reflecting the diminishing marginal returns that characterize maturing portfolios. In the final phase, accumulation approaches a saturation level near the maximum capacity, capturing the upper bound imposed by market constraints. This three-phase behavior reflects an important market reality: small investments can achieve relatively high proportional returns, but as the portfolio grows larger, opportunities to generate further proportional gains become progressively more limited. The logistic model captures these dynamics explicitly through the $-\frac{rA^2}{K}$ term, which introduces a growth-dampening effect that becomes increasingly significant as the investment value approaches the carrying capacity K.

2.3. Prior Research

Several studies have shown the importance of timing in investments. Vanguard (2024) found that missing the ten best days in the stock market over a 20-year period can reduce returns by up to 50% [6]. However, these studies are more focused on market timing rather than systematic comparisons between contribution strategies.

In the Indonesian context, studies on pension fund investments have been conducted by several researchers. [10] analyzed asset allocation and portfolio strategies for employer-sponsored pension funds in Indonesia using Structural Equation Modeling and found that asset allocation has a positive effect on portfolio performance. [11] examined the performance of Financial Institution Pension Funds (DPLK) during the 2019–2024 period, showing aggregate returns of 6.09%. These results are consistent with [7] who showed that public pension fund returns in the US are also relatively similar to passive strategies (60/40), confirming that realistic returns for pension fund investments range between 6–7% annually. Research on asset allocation and pension fund risks in Indonesia was also conducted by [10] who found that defined contribution pension funds tend to show higher risk-taking behavior compared to defined benefit schemes.

In the context of sharia finance, several studies have explored the development of sharia pension funds in Indonesia. [13] analyzed the concept and development of sharia pension funds, while [19] conducted a systematic study on the growth and challenges of sharia pension funds. [17] analyzed Good Islamic Pension Fund Governance (GIPFG) in conventional bank pension funds integrated with sharia principles. Research on determinants of inclusion in sharia pension fund participants was also conducted by [14] who found that Return on Investment and Investment to Asset Ratio influence public participation.

From a behavioral finance perspective, extensive research has been conducted on behavioral biases in retirement planning. [20] provided a comprehensive review of lessons from behavioral research for retirement saving, investment, and spending. [21] conducted a systematic review of behavioral biases in retirement planning in Indonesia, identifying cognitive biases (overconfidence bias, representativeness bias) and emotional biases (self-control, regret aversion) as factors affecting investment decisions. [22] analyzed behavioral biases of financial planners in pension funding recommendations.

The present study differentiates itself from prior work in several respects. It provides a systematic numerical comparison of seven retirement investment strategy scenarios spanning three distinct categories (Core Comparison, Robustness, and Trade-off Analysis) with parameters measured and controlled across scenarios. Through this design, the study delivers a quantitative analysis of the trade-off between investment time, contribution size, and rate of return, demonstrating that time is the dominant factor in long-term accumulation even when total contributions are held constant across strategies. The logistic growth model is employed as a modeling framework that provides a more realistic conceptual foundation for representing investment accumulation with diminishing returns, in contrast to the purely exponential models prevalent in the existing literature. Finally, the study integrates an Indonesian perspective by grounding its parameter choices and practical implications in the domestic regulatory environment and financial market context.

3. Methods

This section presents the mathematical model, numerical method, parameter settings, and scenario design used to compare early and late investment strategies.

3.1. Justification for Choosing Logistic Growth Model

The logistic growth model was chosen in this study for four principal reasons that justify its use in the context of long-term investment accumulation. First, unlike exponential models that assume unlimited growth, logistic models consider realistic market capacity limitations [8], [9], [18]. In investment contexts, as investment value grows larger, opportunities to generate proportional returns become more limited, a phenomenon known as diminishing returns. The logistic model captures these dynamics through the $-\frac{rA^2}{K}$ term that slows growth as the investment value approaches the maximum capacity.

Second, empirical evidence shows that investment returns tend to be higher in early accumulation phases and decrease as the portfolio grows larger [8], [9]. This aligns with logistic model characteristics having rapid growth in early phases and saturation in final phases. Pure exponential models cannot capture this phenomenon and tend to overestimate final values for large investments.

Third, logistic models with constant addition C provide a more realistic representation for investments with regular contributions (like monthly pension investments) compared to traditional discrete annuity models [12]. This model allows dynamic analysis of how periodic contributions interact with already accumulated growth.

Fourth, the application of logistic models in financial analysis has been proven by several researchers. [8] used logistic models to analyze the growth of new financial asset ownership in Australia and found that savings account ownership reached saturation levels. Other research by [9] demonstrated logistic accumulation model applications for Lithuania's Gross Domestic Product, proving that logistic models provide more accurate estimates compared to exponential models. [8] also showed that economic logistic models have universal characteristics in modeling economic growth cycles.

At the beginning of investment when $A \ll K$, the term $-\frac{rA^2}{K} \approx 0$, so growth approaches exponential, consistent with conventional compound interest theory. As A increases, the slowing effect becomes more significant, reflecting more complex market realities. This dual behavior, in which the model approximates exponential compounding in early phases while incorporating realistic deceleration as the portfolio matures, makes the logistic model particularly well-suited for long-term pension fund modeling, where both the initial growth phase and eventual capacity constraints must be considered.

3.2. Mathematical Model

Logistic growth models with regular contributions are represented as differential equations:

$$\frac{dA}{dt} = r_m A \left(1 - \frac{A}{K}\right) + C, \qquad r_m = \frac{r}{12} \tag{1}$$

where $A(t)$ is the investment value at time t with t measured in months (Rupiah), r is the annual rate of return (e.g.,

$r = 0.06$ for 6% per year), $r_m = r/12$ is the monthly rate of return derived from r, K is the scaling parameter that regulates growth slowdown (Rupiah), and C is the discrete monthly contribution amount (Rupiah).

Regarding the treatment of C in the numerical implementation, the differential equation has C with units of Rupiah/month, interpreted as a continuous inflow rate in the continuous model. In the RK4 numerical implementation with $\Delta t = 1$ month, C is treated as a monthly contribution: each time step of 1 month adds exactly C Rupiah to the investment. This yields the same effect as monthly discrete deposits, with the contribution accumulating as $\sum_{i=1}^{n} C = C \cdot n$ over n months. For convergence testing with different Δt values, C remains constant (not scaled by $1/\Delta t$). The RK4 method maintains consistency across different time steps through its 4th-order accuracy, as confirmed by the convergence test in Table 1.

The model comprises three distinct terms. The term $r_m A$ represents proportional growth from investment value, capturing the compounding growth mechanism. The term $-\frac{r_m A^2}{K}$ is the growth dampening factor that regulates the slowdown of growth; parameter K is not a maximum accumulation limit but rather a scaling parameter that determines when the slowdown becomes significant. Because of the constant contribution term C, the investment value can exceed K and continue growing, albeit at a progressively slower rate. The final term $+C$ represents the addition from regular monthly deposits.

3.3. Analytical Benchmark: Exponential-Annuity Model

To establish a benchmark for comparison, we first consider the standard exponential growth model with periodic contributions (the classical annuity model). This model assumes unlimited growth without capacity constraints:

$$\frac{dA}{dt} = r_m A + C, \qquad A(0) = 0$$

This linear first-order ODE has a closed-form analytical solution:

$$A(t) = \frac{C}{r_m}\left(e^{r_m t} - 1\right)$$

For monthly contributions over n months with monthly rate $r_m = r/12$:

$$A(n) = \frac{C}{r_m}\left(e^{r_m n} - 1\right) \tag{2}$$

This analytical solution provides a benchmark to evaluate the logistic model's behavior, particularly showing how the logistic model approaches the exponential solution when $A \ll K$. By comparing the logistic model results against this closed-form benchmark, we can quantify the deviation introduced by the logistic damping factor and verify that the numerical implementation is consistent with known analytical results in the limiting case where capacity constraints are negligible.

3.4. Numerical Method: 4th-order Runge-Kutta

While Eq. (1) is of Riccati type and admits an analytical solution, we use the 4th-order Runge-Kutta (RK4) numerical method [23] for practical reasons:

$$A_{n+1} = A_n + \frac{\Delta t}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

where $A_n = A(t_n)$ is the investment value at month n, and coefficients $k_1, \ldots, k_4$ are defined as:

$$\begin{aligned} k_1 &= f(A_n) \\ k_2 &= f\left(A_n + \frac{\Delta t}{2} k_1\right) \\ k_3 &= f\left(A_n + \frac{\Delta t}{2} k_2\right) \\ k_4 &= f(A_n + \Delta t \cdot k_3) \end{aligned}$$

with function f defined consistently with the model equation:

$$f(A) = r_m A\left(1 - \frac{A}{K}\right) + C, \qquad r_m = \frac{r}{12}$$

where r_m is the monthly rate of return, K is the carrying capacity, and C is the monthly deposit amount. Each coefficient k_i has units of Rupiah/month, so the multiplication $\frac{\Delta t}{6}(\cdots)$ with $\Delta t = 1$ month produces value additions in Rupiah units, consistent with the units of A_n. For convergence testing with $\Delta t \neq 1$, the RK4 method automatically maintains consistency: the contribution C remains constant, but the number of time steps adjusts to maintain the same total duration. The 4th-order accuracy of RK4 ensures convergence across different time step sizes, as verified empirically in Table 1.

The RK4 method was chosen for several practical reasons. Implementing RK4 is straightforward and does not require symbolic manipulation or algebraic simplification of complex expressions. The numerical approach also easily accommodates model modifications (such as time-varying r, stochastic terms, or additional state variables) without requiring re-derivation of analytical formulas. Furthermore, RK4 provides 4th-order accuracy (error per step $\sim \mathcal{O}(\Delta t^5)$), remains stable for small time steps, and is computationally efficient. The time step was chosen as $\Delta t = 1$ month to reflect monthly contributions.

3.4.1. Implementation and Computational Environment

Numerical simulations were implemented using Python 3.11 in the Visual Studio Code environment. The implementation relied on NumPy (version 1.26.x) for numerical array operations and vectorization, Matplotlib (version 3.8.x) for data visualization and graph creation, and the OS library for output directory management. All libraries are open-source licensed. Implementation was performed on the Windows operating system.

3.5. Validation and Accuracy of Numerical Methods

To ensure simulation reliability, a series of validation tests was conducted, including a convergence test across multiple time step sizes and an error estimate analysis confirming that the chosen discretization provides adequate numerical accuracy for the investment projection range examined.

3.5.1. Convergence Test

Convergence tests were performed by comparing results at three time steps: 1 month (baseline), 0.5 months, and 0.25 months. The monthly contribution $C = \text{Rp}\,1,000,000$ is held constant across all time steps. As Δt is halved, the number of integration steps doubles while preserving the same total duration of 30 years (360 months). The numerical results are shown in Table 1.

Tabel 1: Convergence Test Results for RK4 Method (Early–6% Scenario, $K = 10^{12}$, $C = \text{Rp}\,1,000,000$)

Time Step	No. of Steps	Final Value (Rp)	Diff vs.$\Delta t = 1$ month (Rp)
1 month	360	1,009,377,036.29	–
0.5 month	720	1,009,377,036.30	+0.0106
0.25 month	1,440	1,009,377,036.30	+0.0112

The absolute differences between time steps are negligibly small–less than 0.02 Rp on a total accumulation of approximately Rp1,009 billion, corresponding to a relative error of roughly 10^{-9}%. This confirms that $\Delta t = 1$ month provides more than sufficient accuracy for investment projection purposes and that the solution has effectively converged.

3.5.2. Error Estimate Analysis

The RK4 method achieves a local truncation error of $\mathcal{O}(\Delta t^5)$ per step and a global (accumulated) error of $\mathcal{O}(\Delta t^4)$ over the entire simulation. These error orders are derived from the Taylor series expansion of the exact solution:

$$\begin{aligned} y(t+\Delta t) &= y(t) + \Delta t\, y' + \frac{\Delta t^2}{2!}\, y'' + \frac{\Delta t^3}{3!}\, y''' \\ &\quad + \frac{\Delta t^4}{4!}\, y^{(4)} + \frac{\Delta t^5}{5!}\, y^{(5)} + \cdots \end{aligned}$$

Since RK4 exactly reproduces all Taylor series terms up to and including order Δt^4, the leading error term per step is of order Δt^5, giving a local truncation error:

$$\epsilon_{\text{local}} = \mathcal{O}(\Delta t^5)$$

Accumulating this error over N steps, where $N = T/\Delta t$ and T is the total simulation duration, yields the global error:

$$\epsilon_{\text{global}} = N \cdot \mathcal{O}(\Delta t^5) = \frac{T}{\Delta t} \cdot \mathcal{O}(\Delta t^5) = \mathcal{O}(\Delta t^4)$$

Rather than estimating the error bound analytically, which requires bounding higher-order derivatives of the solution $y^{(5)}(t)$ and is non-trivial for the logistic model, accuracy is verified empirically through the convergence test reported in Table 1. The convergence test demonstrates that when the time step is refined from $\Delta t = 1$ month to $\Delta t = 0.5$ months and further to $\Delta t = 0.25$ months, the computed final values converge with absolute differences of less than 0.02 Rp (relative error approximately 10^{-11}) across successive refinements. The Richardson convergence ratio of 16.05, as computed from the successive absolute differences, confirms that the RK4 method exhibits the expected 4th-order ($\mathcal{O}(\Delta t^4)$) convergence behavior. This indicates excellent numerical stability and confirms that $\Delta t = 1$ month is sufficiently small for the accuracy required in this analysis.

Time units are maintained consistently throughout this analysis. Time t is measured in months; the annual rate r is converted to a monthly rate as $r_m = r/12$; the time step Δt equals 1 month for all main simulations; and the duration parameter in all scenarios is specified in months (e.g., 360 months = 30 years). This ensures dimensional consistency in the differential equation $\frac{dA}{dt} = r_m A\left(1 - \frac{A}{K}\right) + C$, where all terms have units of Rupiah/month.

3.6. Parameter K Sensitivity Analysis

To understand the role of the scaling parameter K in regulating growth slowdown, sensitivity analysis was performed with variations in K:

Tabel 2: Sensitivity Analysis Results of K for Early-6% Scenario

K	Final Value (Rp)	Efficiency Ratio
10^9 (1 B)	661,173,226	1.84x
5×10^9 (5 B)	910,963,220	2.53x
10^{11} (100 B)	1,004,433,878	2.79x
10^{12} (1 T)	1,009,377,036	2.80x
10^{13} (10 T)	1,009,874,218	2.81x
10^{14} (100 T)	1,009,923,965	2.81x

Results show that for $K \geq 10^{12}$, final values are relatively stable with variations less than 0.1%. The choice of $K = 10^{12}$ (1 trillion) is appropriate as it does not constrain actual dynamics within the simulation range. However, using smaller values of K (for example, $K = 5 \times 10^9$) makes the logistic growth slowdown clearly visible, as the efficiency ratio decreases significantly from 2.80x to 2.53x. This demonstrates that K regulates the slowdown of growth rather than acting as a strict maximum accumulation limit, since the model also includes the constant contribution term C that allows continued growth beyond K.

3.7. Research Scenario Design

A total of seven scenarios were designed in three categories:

3.7.1. Category A: Core Comparison (Equal Total Contributions, Different Durations)

This category is the core of the research. Both scenarios were designed to have identical total contributions (1 million × 360 months = 360 million):

Tabel 3: Category A: Core Comparison

Scenario	Duration	Monthly Contribution	Annual Return
Early-6%	30 years (360 mth)	Rp1,000,000	6%
Late-6%	15 years (180 mth)	Rp2,000,000	6%

This category tests whether the Early-6% final value exceeds the Late-6% value despite equal total contributions, thereby establishing a baseline comparison where only the investment horizon differs. Confirming this outcome would demonstrate that time, rather than contribution magnitude, drives long-term accumulation even when the total capital outlay is held constant.

The difference in monthly contributions between both scenarios (Rp1,000,000 versus Rp2,000,000) was intentionally designed to reflect realistic financial capacity patterns: individuals who start investing earlier are typically in early career phases with lower income, while those who start later have reached more established career phases with higher financial capacity. Equalizing total contributions serves as an experimental control that allows clean isolation of the time variable effect, so observed differences in final values cannot be confounded by different total invested capital.

3.7.2. Category B: Robustness (Consistency Verification at Higher Returns)

This category tests whether results from Category A are consistent when returns increase:

Tabel 4: Category B: Robustness

Scenario	Duration	Monthly Contribution	Annual Return
Early-7%	30 years (360 mth)	Rp1,000,000	7%
Late-7%	15 years (180 mth)	Rp2,000,000	7%

3.7.3. Category C: Trade-off Analysis (Can High Returns Replace Time?)

This category compares three scenarios with different returns to test whether return increases can compensate for investment timing delays:

Tabel 5: Category C: Trade-off Analysis

Scenario	Duration	Monthly Contribution	Annual Return
Early-5%	30 years (360 mth)	Rp1,000,000	5%
Late-6%	15 years (180 mth)	Rp2,000,000	6%
Very Late-8%	10 years (120 mth)	Rp3,000,000	8%

In this category, we evaluate whether the Early-5% strategy (lowest return, longest duration) yields higher final values than both Late-6% and Very Late-8%, despite the latter two strategies having higher annual returns. A finding that Early-5% outperforms Very Late-8% would demonstrate that, within the tested parameter range, time remains the dominant factor in investment accumulation and that higher returns alone cannot fully substitute for a longer investment horizon.

3.8. Model Parameters

The carrying capacity K was chosen to be sufficiently large (10^{12}) so that it does not artificially constrain dynamics within the realistic investment value ranges examined in this study. This ensures that the logistic damping factor $(1 - A/K)$ remains close to unity throughout all simulations, allowing the comparative analysis between early and late strategies to proceed without the confounding influence of premature saturation. The sensitivity analysis in Section 2 confirms that the results are stable for $K \geq 10^{12}$, justifying this parameter choice.

Tabel 6: Numerical Model Parameters

Parameter	Value	Description
Initial investment (A_0)	Rp0	Investment starts from zero
Carrying capacity (K)	Rp10^{12} (1 trillion)	Chosen so that $A/K \ll 1$ within simulation range
Time step (Δt)	1 month	Reflects monthly contribution frequency
Time unit t	Month	$t = 360$ equals 30 years
Monthly rate (r_m)	$r/12$	E.g., $r = 0.06 \Rightarrow r_m =$ 0.005 per month

3.9. Output Metrics

Three main metrics are calculated for each scenario. The first is the final value $A(T)$, representing the total investment value at the end of the investment period. The second is total contributions, defined as the total amount invested during the period. The third is the net efficiency ratio, given by $\text{ER} = \frac{A(T) - \text{Total Contributions}}{\text{Total Contributions}} = \frac{FV - TC}{TC}$, which measures the profit earned per Rupiah invested.

The relationship between the net efficiency ratio and the total return multiplier warrants careful distinction. A net efficiency ratio of 1.80 (or 1.80x) means that for every Rupiah invested, Rp1.80 of profit is earned. The total return multiplier expresses the final value divided by total contributions (e.g., 2.80x), which equals the net efficiency ratio plus 1 (i.e., $1.80 + 1 = 2.80$). Throughout this paper, when results show "efficiency ratio 2.80x," this refers to the total return multiplier (FV/TC ratio), while the profit per Rupiah invested (net efficiency ratio) is 1.80. A higher ratio on either metric indicates a more efficient strategy.

4. Results and Discussion

This section reports the simulation results and discusses how investment duration, contribution size, and return rate affect long-term pension fund accumulation.

4.1. Numerical Simulation Output

Numerical simulations were conducted using Python programming language with RK4 method implementation. Simulation results cover all scenarios from three main categories (Categories A, B, C) as well as extended scenarios.

The exponential model values are computed using equation Eq. (2) with closed-form analytical integration. Differences between the logistic and exponential models are at most 0.075% across all scenarios, confirming that for $K = 10^{12}$ and a maximum accumulation of approximately Rp1 billion, the logistic damping factor $(1 - A/K)$ has only a marginal effect within the tested simulation range ($A/K \approx 10^{-3}$).

Tabel 7: Comparison: Logistic Model vs Exponential-Annuity Model (Analytical Benchmark)

Scenario	Duration (years)	Exponential Model Final Value (Rp)	Logistic Model Final Value (Rp)	Difference (%)
Early-6%	30	1,009,929,492.88	1,009,377,036.29	-0.055%
Late-6%	15	583,841,243.83	583,741,686.00	-0.017%
Early-7%	30	1,228,486,270.72	1,227,564,567.30	-0.075%
Late-7%	15	636,908,954.75	636,772,028.84	-0.021%
Early-5% (Trade)	30	835,605,376.98	835,280,623.44	-0.039%
Very Late-8%	10	551,493,418.46	551,414,017.08	-0.014%

Tabel 8: Results of Seven Investment Scenario Simulations (Logistic Model). The Efficiency Ratio column reports the total return multiplier (FV/TC); the net efficiency ratio (profit per Rupiah invested) equals this value minus 1.

Scenario	Duration (years)	Total Contributions	Final Value (Rp)	Efficiency Ratio
Early-6%	30	360 million	1,009,377,036	2.80x
Late-6%	15	360 million	583,741,686	1.62x
Early-7%	30	360 million	1,227,564,567	3.41x
Late-7%	15	360 million	636,772,029	1.77x
Early-5% (Trade)	30	360 million	835,280,623	2.32x
Late-6% (Trade)	15	360 million	583,741,686	1.62x
Very Late-8%	10	360 million	551,414,017	1.53x

4.2. Growth Rate Analysis

Growth rate is defined as the change in investment value from one year to the next, normalized against the value at the beginning of the period:

$$g(t) = \frac{A_{t+1} - A_t}{A_t} \times 100\%$$

where A_t is the investment value at the end of year t obtained from logistic model simulations.

Tabel 9: Annual Growth Rates at Different Time Points (% per year)

Scenario	Year 5	Year 10	Year 15	Year 25	Year 30
Early-6%	28.98%	14.82%	10.88%	8.10%	7.49%
Late-6%	28.98%	14.82%	10.88%	-	-
Early-7%	29.69%	15.51%	11.61%	8.91%	8.34%
Late-7%	29.69%	15.51%	11.60%	-	-
Early-5% (Trade)	28.28%	14.15%	10.18%	7.33%	6.69%
Late-6% (Trade)	28.98%	14.82%	10.88%	-	-
Very Late-8%	30.41%	16.22%	-	-	-

The growth rate results reveal three notable patterns. First, a consistent decreasing pattern is observed across all scenarios. Growth rates decrease uniformly from approximately 28–30% at year 5 to 10–12% at year 15, and asymptotically approach r_m at the end of the period. This consistency across all scenarios, regardless of differences in r and duration, indicates that the decrease is structurally driven by the investment model with periodic contributions rather than by the specific parameter values chosen.

Second, the decreasing growth rate is primarily driven by what may be termed a contribution dilution effect: the fixed monthly contribution C becomes an increasingly smaller proportion of the growing portfolio value A_t, so:

$$g(t) = \frac{A_{t+1} - A_t}{A_t} \approx r_m + \frac{C}{A_t} \xrightarrow{A_t \to \infty} r_m$$

For example, the Early-6% scenario experiences a decrease from 28.98% at year 5 to 7.49% at year 30, approaching $r_m = r/12 = 0.5\%$ per month, or approximately 6% per year, asymptotically. The logistic damping factor $(1 - A/K)$ contributes only marginally to this decrease given that $A/K \approx 10^{-3}$ in the simulation range, confirming that the observed decreasing pattern is a universal feature of investment models with periodic contributions rather than a phenomenon exclusive to logistic models.

Third, regarding logistic model relevance, although the damping factor effect is marginal in this simulation range, the logistic framework still provides a more realistic conceptual foundation compared to purely exponential models. It explicitly acknowledges that investment growth has an upper limit, and allows direct extensions to scenarios with tighter K as discussed in Section 5.7.

4.3. Quantitative Comparative Analysis

4.3.1. Category A: Core Comparison

The core comparison contrasts the Early-6% and Late-6% strategies under conditions of equal total contributions (Rp360,000,000) and equal annual returns (6%), with the only differing variable being the investment duration: 30 years for the early strategy versus 15 years for the late strategy.

The numerical simulation results show that the Early-6% strategy, with monthly contributions of Rp1,000,000 over 30 years, achieves a final value of Rp1,009,377,036.29, corresponding to a total return multiplier of 2.80x (net efficiency ratio of 1.80). In contrast, the Late-6% strategy, with monthly contributions of Rp2,000,000 over 15 years, yields only Rp583,741,686, with a total return multiplier of 1.62x (net efficiency ratio of 0.62). The difference in final value between the two strategies is Rp425,635,350: early investing produces a 73% higher accumulation despite identical total capital outlay.

These findings provide quantitative evidence that early-stage investment with small but sustained contributions produces a significantly higher terminal accumulation value than late-stage investment with larger but shorter contributions, given that the total principal invested remains the same across both scenarios. This result stems directly from the exponential growth dynamics embedded in the logistic model, where the investment horizon t emerges as the dominant variable driving long-term wealth accumulation. The model reflects this through two interconnected mechanisms. The first is exponential base expansion in the early phase: returns in the initial period are computed on a principal base that grows continuously over time, so each successive compounding interval operates on a larger effective capital, producing a growth trajectory that accelerates non-linearly as t increases. The second is recursive return reinvestment: returns generated in any given period are folded back into the principal, making them subject to further compounding in subsequent periods. This self-reinforcing process ensures that the effective yield grows as a function of t, which explains why the length of the investment horizon matters more than the size of individual contributions.

4.3.2. Category B: Robustness

The robustness analysis compares the Early-7% and Late-7% strategies to test whether the core findings remain consistent at higher rates of return. The Early-7% strategy achieves a final value of Rp1,227,564,567, with a total return multiplier of 3.41x (net efficiency ratio 2.41), while the Late-7% strategy yields Rp636,772,029, with a total return multiplier of 1.77x (net efficiency ratio 0.77). The pattern observed in Category A is fully reproduced: even with a higher 7% annual return, the early strategy significantly outperforms the late strategy.

This consistency confirms that the relative advantage of early investing is not contingent on the specific 6% return used in the core comparison, but is a structural property of long-term compounding dynamics that persists across different return levels. Indeed, Category B produces a pattern fully consistent with Category A: at the higher 7% return level, the early strategy achieves a substantially greater final value than the late strategy, indicating that the early start provides additional compounding cycles that magnify returns regardless of the underlying rate. This robustness against variations in return rates strengthens the generalizability of the study's conclusions. The finding confirms that the superiority of early investing is a structural phenomenon of long-term accumulation dynamics rather than a result contingent on particular parameter choices.

4.3.3. Category C: Trade-off Analysis

The trade-off analysis addresses the critical question of whether high rates of return can compensate for a delayed start in investing. Three scenarios with different durations and returns but identical total contributions are compared: Early-5% (30 years, Rp1,000,000/month, 5% return), Late-6% (15 years, Rp2,000,000/month, 6% return), and Very Late-8% (10 years, Rp3,000,000/month, 8% return).

The simulation results show that Early-5% achieves a final value of Rp835,280,623, with a total return multiplier of 2.32x (net efficiency ratio 1.32). Late-6% yields Rp583,741,686, with a total return multiplier of 1.62x (net efficiency ratio 0.62). Very Late-8% yields Rp551,414,017, with a total return multiplier of 1.53x (net efficiency ratio 0.53). Notably, even with an 8% return, which is three percentage points higher than the early strategy's 5%, Very Late cannot match Early's final value.

Category C represents the strictest test of the hypothesis. If Very Late-8% (highest return, shortest duration) could surpass Early-5% (lowest return, longest duration), then higher returns would be sufficient to compensate for investment timing delays. However, the simulations show that Early-5% still outperforms by a wide margin, even with a return disadvantage of three percentage points. Under the tested scenarios, an 8% annual return was insufficient to compensate for a shorter 10-year investment horizon against a 5% return over 30 years. This demonstrates that, within the parameter range examined, higher returns cannot fully replace the benefits of earlier investment timing. The compounding advantage generated over 30 years cannot be overcome by higher returns alone; time emerges as the dominant factor in long-term investment accumulation because compound interest from three decades provides exponential benefits that higher rates of return on a compressed timeline cannot replicate.

4.4. Practical Implications

The results of this study carry direct implications for individual financial decision-making. For young individuals, the central message is that it is better to begin investing now with small, manageable amounts than to postpone investing in anticipation of being able to contribute larger sums later. Young professionals, even those with modest current salaries, should recognize that allocating even a small portion of income toward pension investments yields exponential long-term benefits owing to the extended compounding horizon. Furthermore, the findings caution against an overemphasis on return-chasing strategies; specifically, focusing on securing higher returns (e.g., through stock index funds versus bonds) is less effective than maintaining consistent contributions over a sufficiently long investment duration. Each year of delay in starting investments carries a significant opportunity cost that becomes progressively more difficult to overcome, reinforcing the importance of early action in pension fund planning.

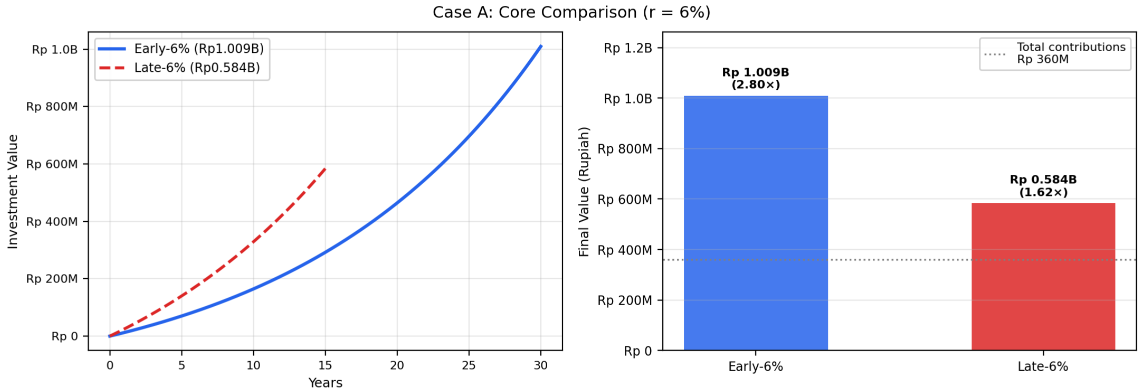
4.5. Theoretical Contributions

This study makes three principal theoretical contributions. First, it applies logistic growth models to the context of pension fund planning with periodic contributions, demonstrating that such models can represent long-term investment accumulation more realistically than purely exponential approaches, particularly in scenarios where diminishing returns apply. Second, the systematic scenario design–spanning core comparison, robustness, and trade-off analyses–enables clean isolation of each variable's influence, showing how investment duration, contribution size, and rate of return interact within a non-linear growth framework. Third, the study provides quantitative evidence on the relative importance of these factors, demonstrating that, under the tested parameter ranges, time exerts a greater influence than contribution size, which in turn outweighs the rate of return in determining final accumulation value.

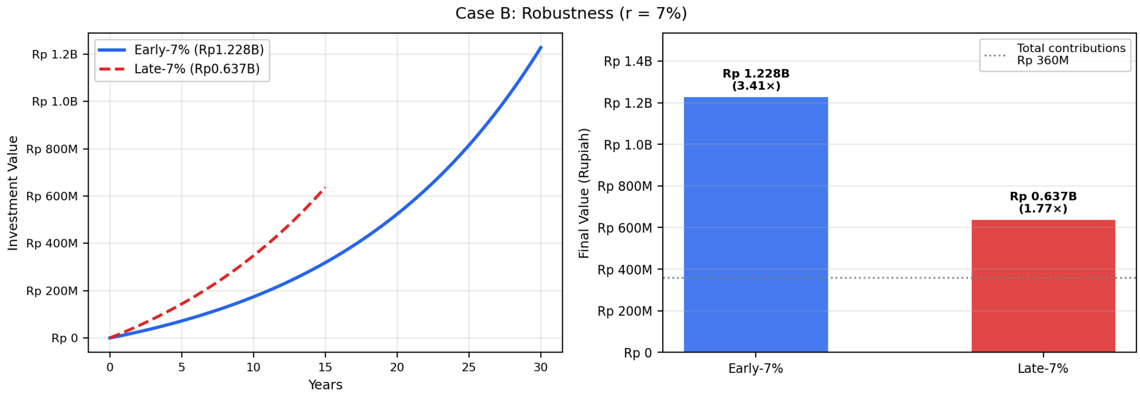
4.6. Limitations of the Study

Although this study provides valuable insights, several limitations should be acknowledged. First, the model assumes fixed rates of return of 6–7% annually, whereas actual investment returns fluctuate from year to year. Sensitivity analysis incorporating stochastic returns would be necessary to assess the robustness of the findings under more volatile market conditions. Second, the simulations assume consistent monthly contributions without any gaps, which may not reflect real-world scenarios where individuals may face periods of unemployment or reduced income that interrupt their contribution schedules.

Third, the model does not account for inflation, which in practice erodes the real purchasing power of nominal pension fund accumulations. A model incorporating inflation-adjusted returns would provide a more complete picture of retirement income adequacy. Similarly, the model neglects the effect of



Gambar 1: Comparison of Early-6% and Late-6% with identical total contributions of Rp360 million. Early-6% (30 years) yields Rp1,009 billion (ratio 2.80x), while Late-6% (15 years) only yields Rp583.7 million (ratio 1.62x). Early investment outperforms by 73% even though total invested money is identical.



Gambar 2: Comparison of Early-7% and Late-7% with equal total contributions (Rp360 million). Early-7% yields Rp1.2 billion (ratio 3.41x), Late-7% yields Rp636.8 million (ratio 1.77x). Results consistent with Category A despite higher returns.

taxes on investment returns; in many jurisdictions, capital gains and dividend income are subject to taxation, which reduces net returns. An after-tax analysis would be needed to evaluate the practical significance of the findings for investors in different tax regimes.

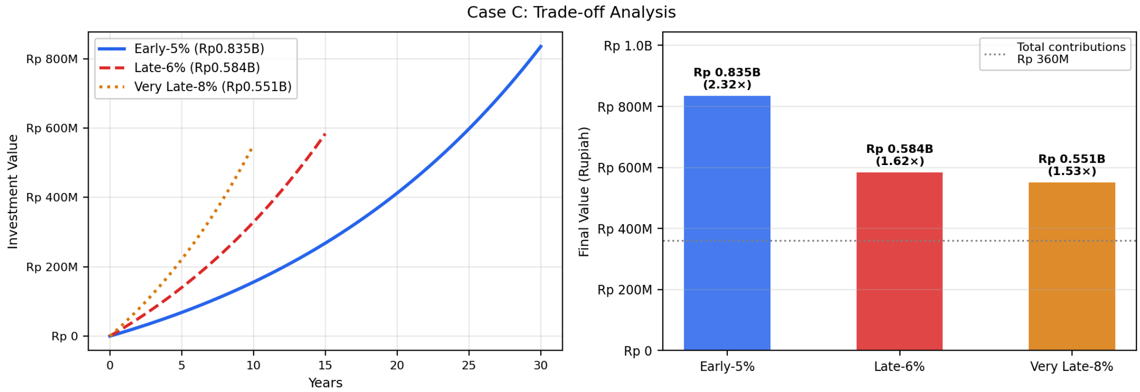
Finally, the logistic damping effects are marginal within the simulation range examined. With a maximum accumulation value of approximately Rp1 billion and a carrying capacity of $K = 10^{12}$, the ratio $A/K \approx 10^{-3}$, so the damping factor $(1 - A/K) \approx 0.999$ throughout the simulation. Consequently, the contribution of logistic effects to the final numerical results is small compared to conventional exponential-annuity models. Nevertheless, the logistic framework retains value in two respects: it provides a conceptually more realistic mathematical foundation by explicitly acknowledging the existence of accumulation capacity limits, and it serves as a foundation for extensions to scenarios with lower K, such as instruments with regulatory investment limits or portfolios with constrained capacity–a promising direction for future investigation. For the comparative analysis between scenarios in this study, model consistency is maintained because all scenarios use the same value of K.

4.7. Future Research Directions

This study opens several directions for further investigation. One natural extension is the incorporation of stochastic models, such as geometric Brownian motion, to capture year-to-year variability in returns and assess the robustness of the findings under uncertain market conditions. A broader sensitivity analysis that expands the parameter ranges–for example, returns from 3% to 12%, investment durations from 5 to 40 years, and a wider spectrum of carrying capacity values–would further delimit the conditions under which early investing maintains its advantage.

Another important direction is the integration of inflation and taxation into the model, enabling analysis of real (inflation-adjusted) after-tax returns that more closely reflect actual retirement outcomes. Dynamic optimization using control theory also presents a promising avenue: rather than comparing fixed contribution schedules, future work could determine the optimal allocation of contributions over time that maximizes terminal wealth under various constraints.

The exploration of tighter carrying capacity values warrants specific attention. Future research could investigate financially meaningful lower values of K, such as those derived from pension contribution regulatory limits (e.g., DPLK contribution caps per OJK regulations), multiples of individual annual income, or empirical saturation data from mutual fund portfolios in Indonesia. Such investigations would allow the logistic damping effects to manifest more substantially at the numerical level, strengthening the differentiation of logistic models from conventional exponential benchmarks. Finally, empirical validation of the model against historical stock and bond market return data would provide an important reality check, testing whether the model's projections align with



Gambar 3: Trade-off between Early-5%, Late-6%, and Very Late-8% with equal total contributions of Rp360 million. Early-5% (30 years) yields Rp835.3 million (ratio 2.32x). Very Late-8% (10 years) only yields Rp551.4 million (ratio 1.53x). High returns cannot compensate for timing delays.

observed long-term investment outcomes.

5. Conclusion

This study set out to answer a fundamental question in pension fund planning: is starting investments earlier really more important than making larger contributions or achieving higher returns? Through numerical simulation of logistic growth models using the 4th-order Runge-Kutta method across seven systematically designed scenarios, three principal findings emerge.

First, time is the dominant factor in long-term investment accumulation. Early investing with small but sustained contributions over long periods yields significantly higher final values compared to late investing with large contributions over short periods, even when the total capital outlay is held constant. In the core comparison, a 30-year early strategy with a 6% return achieved Rp1.009 billion (total return multiplier 2.80x), outperforming an otherwise identical 15-year late strategy by 73%. Second, higher rates of return do not fully compensate for a delayed start. Even with an 8annual return, three percentage points above the early strategy's 5the very late strategy over 10 years could not match the early strategy's terminal value of Rp835 million, demonstrating that the time advantage cannot be replaced by superior returns alone. Third, these conclusions are robust across parameter variations: the relative advantage of early investing persists when the return rate increases from 6confirming that the findings represent a fundamental structural phenomenon of long-term compounding rather than an artifact of specific parameter choices.

For individual pension fund planning, these results carry a clear and actionable message: investing even small amounts early is more important than waiting to invest larger amounts later. Each year of delay represents an opportunity cost that compounds over time, making it progressively harder to catch up. This insight has particular relevance for young individuals, for whom the primary lever for building retirement wealth is not the size of contributions or the selection of high-return instruments, but rather the simple act of starting early and maintaining consistency.

6. Declarations

This section states the authors' contributions, use of AI-assisted tools, competing interests, funding information, acknowledgments, and data availability.

CRediT Authorship Contribution Statement

Nabila Asyiqotur Rohmah: Conceptualization, Methodology, Software, Investigation, Writing - Original Draft, Writing - Review & Editing. **Zahra Nugraheni:** Formal Analysis, Investigation, Writing – Review & Editing. **Mohamad Nur Fauzi:** Software, Validation, Visualization, Writing - Review & Editing.

Declaration of Generative AI and AI-assisted technologies

During the preparation of this manuscript, the authors used AI-assisted tools, including Kilocode integrated in Visual Studio Code, to assist with English translation and LaTeX formatting. These tools were used solely for language refinement and document structuring. The authors take full responsibility for the content, accuracy, and originality of the manuscript.

Declaration of Competing Interest

The authors declare no competing interests.

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Data Availability

This study is based entirely on numerical simulation. The simulation parameters and Python code used to generate all reported results are available from the corresponding author upon reasonable request.

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