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Research Article

Predicting the Drying Time of Banana Slices with Fick's Diffusion Equation

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Abstract. Knowing how long a product must stay in the dryer is a basic question in food processing. In the falling-rate period of drying, introductory unit-operations courses offer two quick answers: a linear drying-rate model and the rule that drying time grows with the square of the slice thickness. The two answers do not agree. When the slice is twice as thick, the first predicts a drying time twice as long and the second predicts four times as long. This paper uses Fick's second law of diffusion to settle the question for a typical banana-slice dryer. The model includes a surface condition taken directly from the textbook drying-rate law, so that both quick rules appear as special cases of one equation, linked by a single number, the Biot number. The equation is solved by separation of variables (Fourier series) and checked independently with the Crank–Nicolson finite-difference method; the two solutions differ by less than 0.0001. For the banana example, the diffusion model predicts a falling-rate period of 10.1 h, compared with 3.6 h from the linear model and 6.7 h from the thickness rule, so within the model both quick rules underestimate the time. Doubling the slice thickness from 5 to 10 mm increases the predicted time about 3.3 times, which lies between the two quick rules. A simple estimate, obtained by adding the two quick-rule times, stayed within about 4% of the full model over the range of parameters tested. These are model predictions based on textbook parameters and still need to be checked against measured drying curves.

Keywords: Food Drying; Fick's Second Law; Fourier Series; Biot Number; Crank–Nicolson Method; Banana

1. Introduction

Drying is one of the oldest ways of preserving food. Removing water slows microbial growth and chemical deterioration and makes the product lighter and cheaper to transport [1, 2]. Dried banana slices and banana chips are popular agroindustrial products in Indonesia, and many small and medium producers dry them in simple tray dryers like the one in Figure 1(a). Drying is also energy-intensive. Every extra hour in the dryer costs fuel or electricity, while removing the product too early leaves it too moist to store safely. A reliable estimate of the drying time is therefore a practical engineering need.

A typical drying curve has two stages [5, 6]. In the *constant-rate period*, the surface of the slice is still wet, and water evaporates as it would from an open dish. Once the moisture content falls below a *critical moisture content* X_c, the surface dries out, and water has to travel from the inside of the slice to the surface. This second stage is the *falling-rate period*. It usually takes longer than the first, even though it removes less water, so it dominates the total drying time.

In the unit-operations course (*Satuan Proses*) taught to agroindustrial technology students, the falling-rate period is handled with two quick tools [7]:

1. a **linear model**, in which the drying rate falls in proportion to the moisture content; and
2. a **thickness rule**, $t \sim L^2/D$, which says that the drying time grows with the square of the slice thickness L, where D is the diffusivity of water in the product.

Both tools are useful, but they give different answers to the same simple question: *if the slices are cut twice as thick, how much longer does drying take?* As shown in Section 2, the linear model answers "twice as long", whereas the thickness rule answers "four times as long". A student or an engineer who uses both tools is left unsure which one to trust.

This paper uses mathematics to resolve the conflict. We describe the movement of water inside the slice with Fick's second law of diffusion, a partial differential equation (PDE) that is standard in food engineering [8, 9] and has been used successfully for banana [10, 11]. The main idea is to choose the surface condition of the PDE so that it matches the textbook linear model. This makes both quick tools special cases of the same model and shows when each of them is reliable.

The mathematical tools used here (Fick's law, Fourier series, and the Crank–Nicolson method) are standard; the contribution of this paper lies in how they are combined and applied. Earlier banana studies such as Karim and Hawlader [10, 12] solve diffusion models to fit or predict measured drying curves, but they do not relate the result to the two quick tools that students learn first. Here, in contrast:

(a) the surface condition is taken directly from the textbook rate law, so that the linear model and the thickness rule become the two limits of one model, connected by the Biot number;

(b) the model shows how the drying time scales with slice thickness between these two limits; and

(c) a practical estimate, the sum of the two quick-rule times, is proposed, and its accuracy is checked systematically over a stated range of parameters, using code that is supplied with the article.

The specific aims are: (i) to build the diffusion model for a layer of banana slices on a tray; (ii) to solve it analytically with Fourier series and to confirm the solution numerically with the Crank–Nicolson method; (iii) to compare the drying times predicted by the three approaches for the banana example from the course notes; and (iv) to give simple formulas that

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(a) Small electric tray dryer with its door open, showing the stacked mesh trays. Photo: Waz8 [3], CC0.



(b) Thin dried banana slices, the type of product studied in this paper. Photo: Sagisavas [4], CC0.

Figure 1: The engineering problem. Banana slices are spread on the trays of a dryer (a) until they reach the target moisture content (b). How long this takes, and how strongly the time depends on the slice thickness, is the question addressed in this paper.

can be used in practice without solving a PDE.

2. Problem Description

This section first describes the drying case used throughout the paper and then shows how the two textbook tools estimate its falling-rate time. Comparing the two estimates reveals the conflict that motivates the rest of the study.

2.1. Case Study: Drying Banana Slices

We use the worked example from the course notes [7]. Banana slices with 80% moisture (wet basis) are dried on trays to 10% (wet basis). The tray loading is 5 kg of wet product per m^2, and the air conditions give a constant drying rate of $R_c = 1.1$ kg m^{-2} h^{-1}. Converted to a dry basis, the moisture contents are $X_0 = 4.0$, $X_c = 1.5$ and $X_f = 0.111$ kg water per kg dry solid. Each m^2 of tray holds $m_s = 1.0$ kg of dry solid.

Two further values are needed for the diffusion model. The course example gives the tray loading, not the slice thickness, so the layer thickness is estimated from the loading. With a product density of about 1000 kg/m^3, the 5 kg/m^2 layer corresponds to an equivalent thickness of $L = 5$ mm, i.e., the thickness of a continuous layer with the same mass per tray area. The effect of other values of L is examined in Section 5. For the effective diffusivity of water in banana we use the value $D = 10^{-9}$ m^2/s given in the course notes, which is of the same order as the values measured by Karim and Hawlader [10]. All parameters are listed in Table 1.

Table 1: Parameters of the banana-drying case study.

Quantity	Symbol	Value
Initial moisture (dry basis)	X_0	4.0 kg/kg
Critical moisture	X_c	1.5 kg/kg
Final moisture	X_f	0.111 kg/kg
Constant drying rate	R_c	1.1 kg $\mathrm{m}^{-2}\mathrm{h}^{-1}$
Dry solid per tray area	m_s	1.0 kg/m^2
Layer thickness	L	5 mm
Effective diffusivity	D	10^{-9} m^2/s

The constant-rate period is easy to calculate. The tray must lose $m_s(X_0 - X_c) = 2.5$ kg of water per m^2 at 1.1 kg m^{-2} h^{-1}, which takes $t_c = 2.27$ h. The open question is how long the falling-rate period lasts, i.e., how long it takes to go from $X_c = 1.5$ to $X_f = 0.111$.

2.2. Two Quick Answers from the Textbook

Linear model (LM). The drying rate is assumed to fall in proportion to the average moisture content $\bar{X}$:

$$R = R_c \frac{\bar{X}}{X_c}. \tag{1}$$

A water balance on the tray gives $m_s\, d\bar{X}/dt = -R$. Solving this ordinary differential equation gives an exponential decrease of moisture, $\bar{X} = X_c\, e^{-kt}$ with rate constant $k = R_c/(m_s X_c)$, and the falling-rate time is

$$t_{\mathrm{LM}} = \frac{m_s X_c}{R_c} \ln \frac{X_c}{X_f}. \tag{2}$$

For the banana example this gives $t_{\mathrm{LM}} = 3.55$ h.

Thickness rule. For diffusion, the characteristic time is $t \sim L^2/D$ [7, 8]. With $L = 5$ mm and $D = 10^{-9}$ m^2/s this gives $L^2/D = 6.9$ h. This is only an order of magnitude, because the rule does not say how dry the product is at that time.

The conflict. The tray loading is proportional to the thickness, $m_s = \rho_s L$, where ρ_s is the mass of dry solid per unit volume. Equation (2) therefore predicts $t_{\mathrm{LM}} \propto L$, so twice the thickness means twice the time. The thickness rule predicts $t \propto L^2$, so twice the thickness means four times the time. The two rules cannot both be right, and the difference matters when a producer chooses how thick to cut the slices.

3. Mathematical Model

To resolve this conflict, we now build a single model that describes how water moves inside the banana layer. We first state the simplifying assumptions, then write the governing equation with its boundary conditions, rewrite it in dimensionless form, and finally show how the model contains both textbook tools as special cases.

3.1. Assumptions

We model the layer on the tray as a flat slab of thickness L (Figure 2). Water moves only in the vertical direction x, from the tray ($x = 0$) to the exposed surface ($x = L$). We assume that [5, 8]:

(A1) water moves inside the slab by diffusion, with a constant effective diffusivity D;

(A2) the slab does not shrink;

(A3) at the start of the falling-rate period, the moisture content is the same everywhere and equal to X_c; and

(A4) the equilibrium moisture content is zero, as in the course example; and

(A5) drying takes place from the top surface only, so no water crosses the bottom face of the layer.

Assumption (A5) describes *one-sided drying*. It applies to a densely loaded tray or a tray lined with a sheet, where the product dries mainly from the top, and it is consistent with the course example, in which the drying rate is given per unit tray area. Mesh trays such as those in Figure 1(a) also let some air reach the bottom of the slices. If both faces of a slice dry equally, the same equations apply with L taken as *half* of the slice thickness, because the middle plane of the slice then plays the role of the impermeable tray.

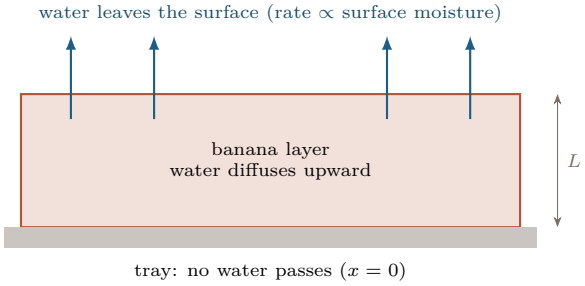


Figure 2: Model of the banana layer on a tray. Water diffuses upward through the layer and evaporates at the top surface; the tray at the bottom is impermeable.

3.2. Governing Equation

Let $X(x, t)$ be the local moisture content (dry basis). Fick's second law states that

$$\frac{\partial X}{\partial t} = D\,\frac{\partial^2 X}{\partial x^2}, \qquad 0 < x < L,\ t > 0. \tag{3}$$

In words, moisture decreases fastest where the moisture profile is most curved. Two boundary conditions describe the two faces of the layer:

- *Tray ($x = 0$):* no water crosses the tray, so $\partial X/\partial x = 0$.
- *Surface ($x = L$):* we apply the textbook rate law (1) *locally*, i.e., to the moisture at the surface instead of the average moisture. The water flux arriving at the surface by diffusion must equal the flux leaving it:

$$-\rho_s D\,\frac{\partial X}{\partial x}(L, t) = \frac{R_c}{X_c}\,X(L, t). \tag{4}$$

Condition (4) is called a *Robin* (or convective) boundary condition. At the critical point the surface moisture is X_c, so the flux in (4) equals R_c. The drying rate therefore continues smoothly from the constant-rate period, which is what is observed in practice.

3.3. Dimensionless Form and the Biot Number

Engineers often remove units from an equation to see which combinations of parameters really matter. We define

$$\xi = \frac{x}{L}, \qquad \tau = \frac{Dt}{L^2}, \qquad u = \frac{X}{X_c}, \qquad \mathrm{Bi} = \frac{R_c L}{\rho_s X_c D} = \frac{kL^2}{D}.$$

Here ξ is the relative position, τ is the *dimensionless time* (Fourier number), u is the moisture relative to X_c, and Bi is the mass-transfer *Biot number*. The problem becomes

$$\begin{aligned} &u_\tau = u_{\xi\xi}, && 0 < \xi < 1,\\ &u_\xi(0, \tau) = 0, \quad u_\xi(1, \tau) = -\mathrm{Bi}\,u(1, \tau), &&\\ &u(\xi, 0) = 1. && \end{aligned} \tag{5}$$

What we measure in practice, by weighing the tray, is the average moisture. We therefore track the *moisture ratio*

$$M(\tau) = \int_0^1 u(\xi, \tau)\,d\xi = \frac{\bar{X}}{X_c}.$$

The target of the banana example is $M^* = X_f/X_c = 0.074$.

The Biot number compares two resistances to drying: the *internal* resistance (water moving through the slice) and the *external* resistance (water leaving the surface into the air). Table 2 summarises its meaning. For the banana example, $k = R_c/(m_s X_c) = 2.04 \times 10^{-4}$ s^{-1} and Bi = 5.09, so both resistances matter.

Table 2: Physical meaning of the Biot number.

Bi	What controls drying
$\mathrm{Bi} \ll 1$	the air (external resistance); the moisture inside stays nearly uniform
$\mathrm{Bi} \approx 1$–10	both the air and the slice
$\mathrm{Bi} \gg 1$	diffusion inside the slice; the surface is almost dry immediately

3.4. Three Models Compared

Model (5) contains the two textbook tools as special cases. In the rest of the paper we compare three models (Table 3):

- **LM** (linear model): assumes a flat moisture profile inside the slice, so the surface moisture equals the average moisture.
- **SDM** (simple diffusion model): assumes that the surface dries instantly, $u(1, \tau) = 0$. This corresponds to $\mathrm{Bi} \to \infty$ and leads to the thickness rule.
- **FDM** (full diffusion model): model (5) with the actual Biot number.

Table 3: The three models compared in this paper.

Model	Inside the slice	At the surface
LM	no gradient (flat)	rate law (1)
SDM	diffusion (3)	dry at once ($X = 0$)
FDM	diffusion (3)	rate law (4)

4. Solution Method

The model of Section 3 is solved in two independent ways. The analytical route uses separation of variables, which leads to an eigenvalue equation, a Fourier series solution, and a simple drying-time formula. The numerical route uses the Crank–Nicolson method and serves as a check on the analytical result.

4.1. Separation of Variables

The PDE (5) is linear, so it can be solved with the classical method of separation of variables [13, 14]. We look for solutions of the form $u = \phi(\xi)\,T(\tau)$. Substituting into the PDE and dividing by ϕT gives $T'/T = \phi''/\phi$. A function of τ alone can equal a function of ξ alone only if both are equal to the same constant, which we call $-\mu^2$. Hence

$$T(\tau) = e^{-\mu^2\tau}, \qquad \phi'' + \mu^2\phi = 0.$$

The tray condition $\phi'(0) = 0$ gives $\phi = \cos(\mu\xi)$, and the surface condition $\phi'(1) = -\mathrm{Bi}\,\phi(1)$ gives

$$\mu \tan\mu = \mathrm{Bi}. \tag{6}$$

4.2. Solving the Eigenvalue Equation

Equation (6) cannot be solved by hand, but it is easy to solve graphically and numerically. Figure 3 plots the left-hand side $\mu\tan\mu$ together with the horizontal line $\mathrm{Bi} = 5.09$. Every branch of $\mu\tan\mu$ rises from 0 to infinity between $(n-1)\pi$ and $(n-\frac{1}{2})\pi$, so it crosses the line exactly once. Each crossing is one eigenvalue μ_n. Because every root lies in a known interval, the bisection method [15] always finds it; we used Brent's method, a faster variant of bisection. For the banana case, $\mu_1 = 1.318$, $\mu_2 = 4.042$ and $\mu_3 = 6.918$.

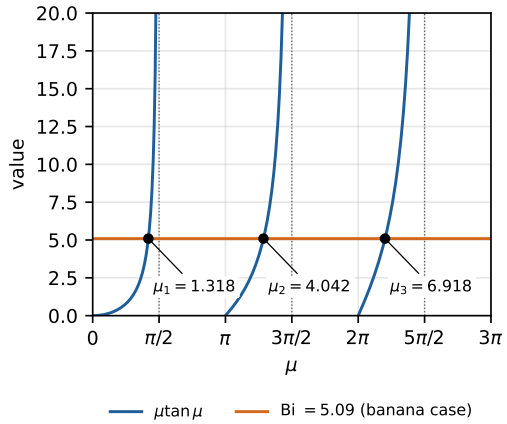


Figure 3: Graphical solution of $\mu\tan\mu = \mathrm{Bi}$. Each crossing of a blue branch with the orange line is one eigenvalue μ_n. The dotted vertical lines are the asymptotes $\mu = (n-\frac{1}{2})\pi$.

4.3. Fourier Series Solution

The full solution is a sum of all such modes, with coefficients chosen so that $u = 1$ at $\tau = 0$:

$$u(\xi,\tau) = \sum_{n=1}^{\infty} c_n \cos(\mu_n\xi)\, e^{-\mu_n^2\tau}, \quad c_n = \frac{4\sin\mu_n}{2\mu_n + \sin 2\mu_n}. \tag{7}$$

Integrating over the thickness gives the moisture ratio

$$M(\tau) = \sum_{n=1}^{\infty} A_n\, e^{-\mu_n^2\tau}, \quad A_n = \frac{2\,\mathrm{Bi}^2}{\mu_n^2\left(\mu_n^2 + \mathrm{Bi}^2 + \mathrm{Bi}\right)}. \tag{8}$$

The coefficients c_n follow from the orthogonality of the functions $\cos(\mu_n\xi)$ [13], and the expression for A_n is simplified using (6). For the SDM ($\mathrm{Bi}\to\infty$), the eigenvalues become $\mu_n = (2n-1)\pi/2$ and $A_n = 8/[(2n-1)^2\pi^2]$, which is the classical result of Crank [8].

4.4. A Simple Drying-Time Formula

The terms in (8) decay very quickly, because μ_2^2 is much larger than μ_1^2. Once drying has progressed a little, only the first term matters, $M \approx A_1 e^{-\mu_1^2\tau}$. Solving for the time gives a practical formula:

$$t_f \approx \frac{L^2}{\mu_1^2 D}\,\ln\frac{A_1}{M^*}. \tag{9}$$

For the SDM, $A_1 = 8/\pi^2$ and $\mu_1 = \pi/2$, which gives the well-known formula $t_f = \frac{4L^2}{\pi^2 D}\ln\frac{8}{\pi^2 M^*}$.

The accuracy of this formula can be estimated directly. All A_n are positive and sum to 1, and all higher terms decay at least as fast as $e^{-\mu_2^2\tau}$. For the SDM this bounds the relative error by $(\pi^2/8 - 1)\,e^{-2\pi^2\tau}$, which is below 1% once $\tau \geq 0.16$. In practical terms, the SDM formula is accurate to within 1% once about 45% of the water to be removed in the falling-rate period has already left the product (Figure 4). For the FDM of the banana case the gap $\mu_2^2 - \mu_1^2 = 14.6$ is somewhat smaller than the SDM value $2\pi^2 \approx 19.7$, so the higher terms fade slightly more slowly, but they are still negligible at the end of drying, where $\tau_f = 1.447$ (Section 5). Since more than 90% of this water must be removed in the banana example, formula (9) is accurate enough for both models.

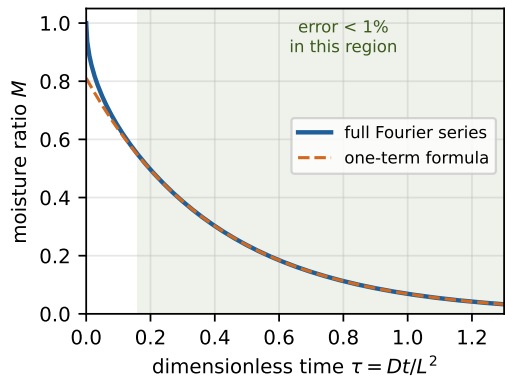


Figure 4: Full Fourier series compared with the one-term formula (SDM). In the shaded region the difference is below 1%.

4.5. Numerical Check: Crank–Nicolson Method

To check the analytical solution independently, we also solve (5) numerically with the Crank–Nicolson finite-difference method [16]. Unlike the Fourier series, this method can easily be extended to cases where D varies or the slice shrinks. The layer is divided into J equal intervals of width $\Delta\xi = 1/J$, and time advances in steps $\Delta\tau$. At each step, the second

derivative is replaced by the average of the usual three-point difference at the old and the new time levels. This produces a tridiagonal linear system, which is solved at every step. The boundary conditions are imposed with "ghost" points just outside the layer.

The Crank–Nicolson method is stable for any time step [17]. Our initial condition ($u = 1$ everywhere) does not satisfy the surface condition exactly, which can cause small oscillations at the start. Following Rannacher [18], we therefore replace the first step by a few fully implicit (backward Euler) steps. The procedure is summarised in Algorithm 1.

Algorithm 1: Crank–Nicolson Solver

Input: Bi, number of intervals J, time step $\Delta\tau$, end time
Output: moisture ratio M at each time step

1. Set $\Delta\xi = 1/J$, $r = \Delta\tau/\Delta\xi^2$ and build the difference matrix A (with ghost points)
2. Start with $u_j = 1$ at all grid points
3. Do 4 backward-Euler half-steps *// smooth start*
4. **repeat** until the end time:
solve $(\mathsf{I} - \frac{r}{2}\mathsf{A})\,\mathbf{u}^{\text{new}} = (\mathsf{I} + \frac{r}{2}\mathsf{A})\,\mathbf{u}^{\text{old}}$
compute M with the trapezoidal rule
5. **return** the values of M

Table 4 compares the numerical and series results for the banana case at $\tau = 0.2$. Each time the grid is refined by a factor of two, the error falls by a factor of four, so the method is second-order accurate, as expected. With $J = 40$ intervals, the difference is already below 10^{-4}, which confirms the series solution.

Table 4: Crank–Nicolson error at $\tau = 0.2$ for Bi $= 5.09$, with $\Delta\tau = \Delta\xi/2$ (series value $M = 0.646944$).

J	$\Delta\tau$	error	error ratio
10	0.05	1.51×10^{-3}	—
20	0.025	3.62×10^{-4}	4.2
40	0.0125	8.99×10^{-5}	4.0
80	0.00625	2.24×10^{-5}	4.0
160	0.003125	5.60×10^{-6}	4.0

5. Results and Discussion

We now apply the solutions to the banana case. We first compare the drying times predicted by the three models, then explain the differences using the moisture profiles and the Biot number, examine the effect of slice thickness, and finally propose a quick estimate for practical use and discuss the limitations of the model.

5.1. Drying Time of the Banana Slices

Table 5 and Figure 5 compare the three models for the banana example. The constant-rate period (2.27 h) is the same in all models; the difference lies entirely in the falling-rate period. For the FDM, formula (9) with $\mu_1^2 = 1.736$ and $A_1 = 0.912$ gives $\tau_f = 1.447$, which agrees with the full series to four digits. One unit of τ equals $L^2/D = 6.94$ h, so $t_f = 10.05$ h.

The linear model predicts that the slices are dry after about 5.8 h in total, whereas the diffusion model predicts more than

Table 5: Predicted falling-rate and total drying times for the banana slices ($L = 5$ mm, $M^* = 0.074$).

Model	falling-rate (h)	total (h)	falling-rate vs. FDM
LM (textbook)	3.55	5.82	−65%
SDM (L^2/D)	6.74	9.01	−33%
FDM (this work)	10.05	12.32	—

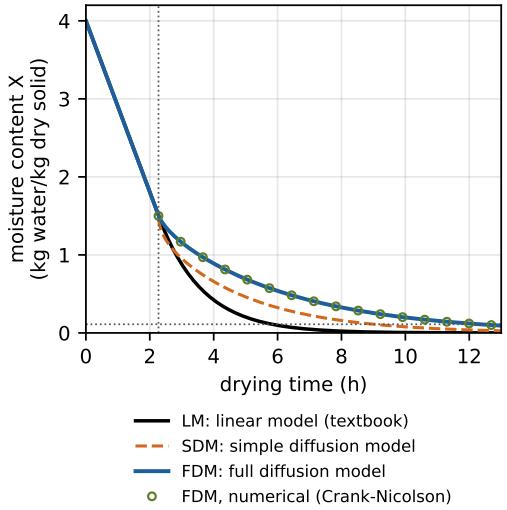


Figure 5: Drying curve of the banana slices. The vertical dotted line marks the end of the constant-rate period, and the horizontal dotted line marks the target moisture X_f. Circles show the Crank–Nicolson solution, which falls on top of the FDM series solution.

12 h. If these predictions are confirmed experimentally, a batch planned with the linear model would leave the slices too moist at the end of the planned drying time.

5.2. Why the Linear Model Is Too Optimistic

The reason becomes clear from the moisture profiles inside the layer (Figure 6), computed from (7); the surface is at 5 mm. Soon after the critical point, the surface becomes much drier than the interior. Because the evaporation rate depends on the *surface* moisture, it is lower than the linear model assumes, since the linear model uses the *average* moisture. This holds at every moment, because the surface is always the driest part of the layer. The linear model therefore always overestimates the drying rate and underestimates the drying time.

The drying-rate curves in Figure 7 show the same effect from another angle. The FDM rate starts at exactly R_c, so it joins the constant-rate period smoothly, and then falls below the straight line of the linear model. The SDM rate is unrealistically high at the start, because it assumes that the surface dries instantly, but it soon drops below the linear model as well.

5.3. The Biot Number Connects the Models

Figure 8 shows the FDM for several Biot numbers on a common time scale kt. When Bi is small (thin slices or slow air), the FDM curve almost coincides with the linear model: the moisture inside stays nearly uniform, which is exactly the assumption of the LM. When Bi is large (thick slices or fast,

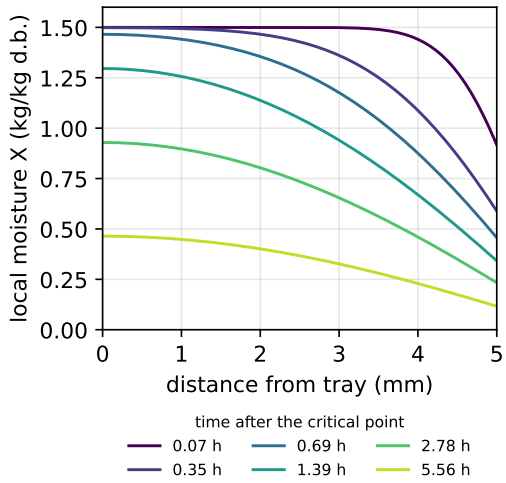


Figure 6: Moisture profiles inside the 5 mm banana layer (FDM) at several times after the critical point. The surface (right) dries much faster than the part near the tray (left).

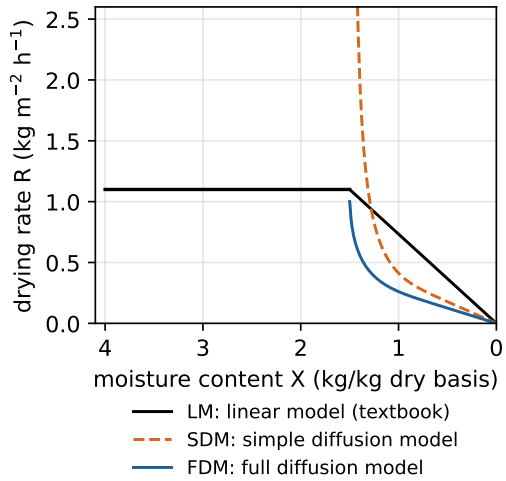


Figure 7: Drying rate as a function of moisture content. Drying proceeds from left to right.

dry air), the FDM approaches the SDM and the thickness rule. In other words:

- the linear model is the *small-Biot limit* of the diffusion model; and
- the thickness rule is the *large-Biot limit*.

For any finite Biot number, the full model lies above both limits, which means it predicts a longer drying time than either quick tool. Table 5 illustrates this for the banana example.

5.4. Effect of Slice Thickness

This section returns to the question posed in the Introduction. We keep the product and the air conditions fixed and change only the layer thickness L. The Biot number then grows in proportion to L. Figure 9 and Table 6 show the result. For very thin layers, the FDM follows the linear model, and the drying time is proportional to L. For thick layers, it follows the thickness rule, and the drying time is proportional to L^2. In between, the drying time grows as L^p, where the exponent p moves gradually from 1 to 2.

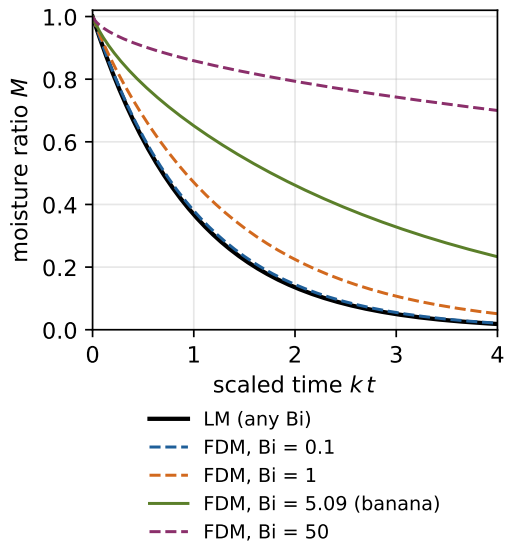


Figure 8: FDM solutions for different Biot numbers, compared with the linear model. As Bi decreases, the FDM approaches the linear model.

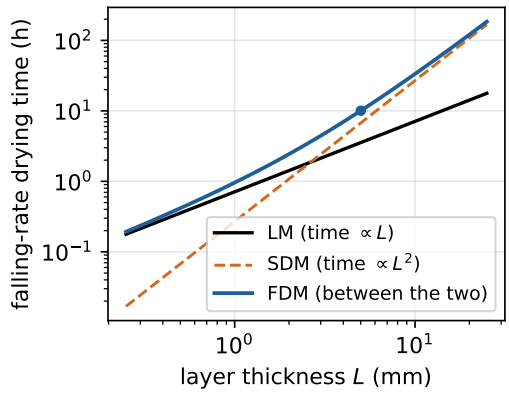


Figure 9: Falling-rate drying time as a function of layer thickness (logarithmic axes). The dot marks the 5 mm banana case. A straight line of slope 1 means time $\propto L$; slope 2 means time $\propto L^2$.

Table 6: Falling-rate drying time (h) for different layer thicknesses. The exponent p gives the local growth of time with thickness (time $\propto L^p$).

L (mm)	Bi	LM	SDM	FDM	p
0.5	0.51	0.36	0.07	0.42	1.15
1.5	1.53	1.07	0.61	1.63	1.36
2.5	2.55	1.78	1.68	3.37	1.49
5	5.09	3.55	6.74	10.05	1.66
10	10.19	7.10	26.95	33.47	1.80
20	20.37	14.20	107.79	120.73	1.89

For the doubling question, going from $L = 5$ mm to $L = 10$ mm increases the predicted falling-rate time from 10.05 h to 33.47 h, a factor of $33.47/10.05 \approx 3.3$, compared with 2 for the LM and 4 for the thickness rule. The exponent p in the table is a *local* quantity: $p = 1.66$ at $L = 5$ mm describes small changes around 5 mm (for example, 10% thicker slices need about $1.1^{1.66} - 1 \approx 17\%$ more time). Over a full doubling the exponent increases from 1.66 to 1.80, which is why the actual factor (3.3) is slightly larger than the estimate $2^{1.66} \approx 3.2$ obtained with the local exponent alone. For a producer this

has a clear practical meaning: cutting thinner slices is the most effective way to shorten drying, but its benefit is somewhat smaller than the textbook L^2 rule suggests.

5.5. A Quick Estimate for Practice

For routine use in production, solving the eigenvalue equation is inconvenient. Table 6 suggests a simpler alternative: the FDM time is close to the *sum* of the LM and SDM times,

$$t_{\mathrm{FDM}} \approx t_{\mathrm{LM}} + t_{\mathrm{SDM}}. \tag{10}$$

This is the same idea as adding resistances in series in heat transfer: the LM time represents the external (air-side) resistance, and the SDM time represents the internal resistance of the slice. For the banana example, (10) gives $3.55 + 6.74 = 10.29$ h, only 2.4% above the exact 10.05 h.

The accuracy of (10) was tested systematically. Written in dimensionless time, the three quantities in (10) depend only on Bi and the target M^*: $\tau_{\mathrm{LM}} = \ln(1/M^*)/\mathrm{Bi}$, τ_{SDM} is the root of the SDM series $M(\tau) = M^*$, and the exact τ_{FDM} is the root of (8). Both roots were found with Brent's method using 200–400 series terms. The test grid used 40 values of Bi, equally spaced on a logarithmic scale from 0.01 to 10^4, and 20 values of M^*, equally spaced on a logarithmic scale from 0.001 to 0.1, giving 800 combinations. For each one we computed the relative error $(\tau_{\mathrm{LM}} + \tau_{\mathrm{SDM}} - \tau_{\mathrm{FDM}})/\tau_{\mathrm{FDM}}$. Within this range the estimate always *overestimated* the exact time, by 0.002% to 4.2%; the largest error occurred near $\mathrm{Bi} \approx 3$ with $M^* = 0.001$. A slight overestimate is on the safe side for dryer planning.

This bound holds only within the tested range. An additional check with targets that stop drying early ($0.1 < M^* \le 0.6$, same Bi values) showed that the estimate can then *underestimate* the time by up to about 7%. The accuracy of 4.2% or better therefore applies to $0.01 \le \mathrm{Bi} \le 10^4$ and $0.001 \le M^* \le 0.1$, a range that covers practical drying to a storable moisture content (the banana case has $M^* = 0.074$). The script and the full table of errors are provided as supplementary material. Both terms in (10) need only a calculator: t_{LM} comes from (2) and t_{SDM} from the SDM version of (9).

5.6. Limitations

The model has several simplifications. The diffusivity of water in fruit actually depends on moisture content and temperature, and banana slices shrink considerably during drying [12, 19]. The initial moisture profile at the critical point may also not be perfectly uniform. All parameters were taken from a textbook example rather than measured in a laboratory, and the layer thickness was estimated from the tray loading. The drying times reported here are therefore *model predictions*; they have not been compared with measured drying times, and the actual values for a given dryer may differ. These effects can be added to the Crank–Nicolson solver without changing its structure. The natural next step is to fit D and R_c to measured drying curves of local banana varieties.

6. Conclusion

This paper used Fick's second law to predict the drying time of banana slices in the falling-rate period. A surface condition based on the textbook drying-rate law links the two quick tools taught in unit-operations courses. The linear model and the thickness rule turn out to be the small-Biot and large-Biot limits of the same diffusion model. The model was solved with Fourier series, using a simple graphical and numerical solution of the eigenvalue equation, and checked with a second-order Crank–Nicolson scheme.

For the banana example, with parameters from a course example, the model predicts a falling-rate period of 10.05 h, almost three times the value from the linear model (3.55 h) and one and a half times the value from the thickness rule (6.74 h). Within the model, both quick tools are therefore too optimistic. Doubling the slice thickness from 5 to 10 mm increases the predicted drying time about 3.3 times. The sum of the two quick-rule times is a simple and slightly conservative estimate of the model's drying time, accurate to within about 4% for Biot numbers from 0.01 to 10^4 and target moisture ratios from 0.001 to 0.1. These conclusions rest on the assumptions of Section 3; they should be confirmed with measured drying curves before they are used to design a real dryer. These results show how basic mathematical tools, namely differential equations, Fourier series, and numerical methods, can answer a practical question in agroindustrial processing.

Author Contributions

Najwa Muhammad: Conceptualization; Methodology; Formal Analysis; Software; Visualization; Writing – Original Draft; Writing – Review & Editing.

Declaration of Generative AI Use

A generative AI writing assistant was used to support grammatical editing, and code preparation. All equations, numerical results, and conclusions were checked and verified by the author.

Competing Interests

The author declares no competing interests.

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Data Availability

No experimental data were generated. All parameter values are taken from the cited course example and are listed in full in Table 1. The photographs in Figure 1 are in the public domain (CC0) and were obtained from Wikimedia Commons.

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